7049

BOARD DIPLOMA EXAMINATION, (C-20)

MAY—2023

DME - FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 Hours] [*Total Marks* : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answers should be brief and straight to the point and shall not exceed five simple sentences.

1. If $A = \{-1, 0, 1\}$ and $f : A \to B$ is a function such that $f(x) = x^3$, then find the range of f.

2. Resolve $\frac{x}{(x-1)(x-3)}$ into partial fractions.

3. If $A = \begin{pmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \end{pmatrix}$, then find $A + B$ and $A - B$.

4. Prove that $\frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ} = \tan 57^\circ$

5. Prove that $\frac{1 + \cos 2\theta}{\sin 2\theta} = \cot\theta$

6. Find the modulus of the complex number $(3 + 4i)(2 - 3i)$.

7. Find the distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$.

8. Evaluate $\lim_{x \to 0} \frac{\tan 9x}{\tan 4x}$.

9. Differentiate $\sqrt{x} - \sec x + \log x$ w.r.t. x.

10. If $y = 6x^2 - 12x + 1$, find $\frac{d^2y}{dx^2}$.

PART—B

8×5=40

Instructions : (1) Answer **all** questions.

(2) Each question carries **eight** marks.

(3) Answers should be comprehensive and criterion for valuation is the content but not the length of the answer.

11. *(a)* If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then find A^{-1}.

(OR)

(b) Solve the system of linear equations

$2x - y + 3z = 9$, $x + y + z = 6$, $2x - y + z = 2$ using Cramer's rule.

12. *(a)* Prove that $\cos 55° + \cos 175° + \cos 65° = 0$.

(OR)

(b) Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$.

13. *(a)* Solve $2\sin^2\theta - \sin\theta - 1 = 0$.

(OR)

(b) In any ΔABC, show that $\sum 2bc \cos A = a^2 + b^2 + c^2$

14. *(a)* Find the equation of the circle with (4, 2) and (1, 5) as the two ends of its diameter and also, find its centre and radius.

(OR)

(b) Find the equation of the rectangular hyperbola whose focus is the point (–1, –3), and directrix is the line $2x + y + 1 = 0$.

15. *(a)* If $x = at^2$ and $y = 2at$, then find $\frac{dy}{dx}$.

(OR)

(b) If $u(x, y, z) = \log(x^2 + y^2 + z^2)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2$.

PART—C

10×1=10

Instructions : (1) Answer the following question.

(2) The question carries **ten** marks.

(3) Answer should be comprehensive and the criterion for valuation is the content but not the length of the answer.

16. The sum of two numbers is 6. Find the numbers if the sum of their squares is minimum.

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