Code No: R10102 **R10** SET - 1

I B. Tech I Semester Supplementary Examinations, January - 2020
MATHEMATICS-I
(Com. to All branches)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Solve $y(2x^2y + e^x)dx = (e^x + y^3)dy$. (8M)

   b) A bacterial culture, growing exponentially, increase from 200 to 500 grams in the period from 6a.m. to 9a.m. How many grams will be present at 12 noon? (7M)

2. a) Solve $(D^2 + 16)\, y = e^{-4x}$. (8M)

   b) Solve $D^2\,(D^2 + 4)\, y = 96x^2 + x \sin 2x$ (7M)

3. a) If $x = e^r \sec\theta$, $y = e^r \tan\theta$ prove that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$. (8M)

   b) Investigate the maxima and minima, if any, of the function $f(x) = x^3 y^2 (1 - x - y)$. (7M)

4. a) Trace the curve $x^3 + y^3 + 3axy = 0$. (8M)

   b) Trace the curve $r = a\,(1 - \cos\theta)$. (7M)

5. a) Find volume of the solid that results when the region enclosed by the curve ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (0<b<a) rotates about major axis. (8M)

   b) Find the arc length of the curve $3x^2 = y^3$ between y = 0 and y =1. (7M)

6. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dydx.$ (8M)

   b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)dxdy.$ (7M)

7. a) Find the directional derivative of $xyz^2 + xz$ at (1, 1, 1) in a direction of the normal to the surface $3xy^2 + y = z$ at (0, 1, 1). (8M)

   b) If $\bar{r}$ is the position vector of the point (x, y, z), prove that $div.grad(r^n) = n(n+1)r^{n-2}$. (7M)

8. a) Evaluate $\int_C \overline{F}.dr$ where $\overline{F} = 3xy\bar{i} - y^2\bar{j}$ and C is the curve $y = 2x^2$ in xy-plane from (0, 0) to (1, 2). (8M)

   b) Use Gauss divergence theorem to evaluate $\iint_S (yz\bar{i} + zx\bar{j} + 2zx^2\bar{k}).ds$, where S is the closed surface bounded by the xy - plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane. (7M)
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