

I B. Tech I Semester Supplementary Examinations, May/June - 2019
MATHEMATICS-I
 (Com. to All branches)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions
 All Questions carry **Equal** Marks

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1. a) Solve $x \frac{dy}{dx} + y = x^3 y^6$. (8M)
 - b) Suppose that an object is heated to 300°F and allowed to cool in a room whose air temperature is 80°F , if after 10 minutes the temperature of the object is 250°F , what will be its temperature after 20 minutes. (7M)
 2. a) Solve $(D^3 - 4D + 13)y = e^{2x}$. (8M)
 - b) Solve $(D^2 + 3D + 2)y = 2\cos(2x + 3) + x\cos x$. (7M)
 3. a) Find the maximum and minimum values of the function $f(x) = x^5 - 3x^4 + 5$. (8M)
 - b) If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (7M)
 4. a) Trace the curve $9ay^2 = x(x - 3a)^2$. (8M)
 - b) Trace the curve $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$. (7M)
 5. a) Find the surface area of the solid generated by revolving the arc of the parabola $x^2 = 12y$, bounded by its latus rectum about y-axis. (8M)
 - b) Find the perimeter of the loop of the curve $3ay^2 = x(x - a)^2$. (7M)
 6. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2 y^2 (x + y) dy dx$. (8M)
 - b) By changing the order of integration, evaluate $\int_0^1 \int_1^{2-x} xy dx dy$. (7M)
 7. a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P=(1,2,3)$ in the direction of the line PQ where $Q = (5,0,4)$. (8M)
 - b) Prove that $\text{grad}(\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \nabla)\vec{a} + (\vec{a} \cdot \nabla)\vec{b} + \vec{b} \times \text{curl} \vec{a} + \vec{a} \times \text{curl} \vec{b}$. (7M)
 8. a) Find the work done by $\vec{F} = (2x - y - z)\vec{i} + (x + y + z)\vec{j} + (3x - 2y - 5z)\vec{k}$ along a curve C in the xy - plane given by $x^2 + y^2 = 9$, $z = 0$. (8M)
 - b) Use Gauss divergence theorem to evaluate $\iiint_S (yz^2 \vec{i} + zx^2 \vec{j} + 2z^2 \vec{k}) \cdot d\vec{s}$, where S is the closed surface bounded by the xy - plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane. (7M)