

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

* * * * *

1. (a) Solve $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$.
(b) Find the orthogonal trajectories of $r^2 = a \sin 2\theta$. [8+7]
2. (a) Solve $(D^3 + 2D^2 + D)y = x + \sin 2x$.
(b) Solve $(D^3 - 4D^2 - D + 4)y = e^{3x} \cos 2x$. [8+7]
3. (a) Show that $U = x^2 e^{-y} \cosh z$, $V = x^2 e^{-y} \sinh z$, $w = x^2 + y^2 + z^2 - xy - yz - zx$ are functionally dependent. If dependent find the relationship between them.
(b) Investigate the maxima and minima, if any, of the function $f(x) = x^3 y^2 (1 - x - y)$. [8+7]
4. Trace the curve $x^3 + y^3 + 3axy = 0$. [15]
5. (a) Find the length of the arc of the curve $y = \log \left(\frac{e^x - 1}{e^x + 1} \right)$ from $x = 1$ to $x = 2$.
(b) Find the volume of the solid that results when the region enclosed by the curves $xy = 1$, x - axis and $x = 1$ rotated about x - axis. [8+7]
6. (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2} dy dx$.
(b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y) dx dy$. [8+7]
7. (a) Find the angle between the normal to the surface $x^2 = yz$ at the points $(1,1,1)$ and $(2,4,1)$.
(b) Find the constants a, b, c so that the vector $\vec{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. Also find ϕ (scalar potential). [8+7]



8. (a) Find the work done by $\vec{F} = (2x - y - z)\vec{i} + (x + y + z)\vec{j} + (3x - 2y - 5z)\vec{k}$ along a curve C in the xy - plane given by $x^2 + y^2 = 9, z = 0$.
- (b) Evaluate $\int_V \vec{F} \cdot dV$ when $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and V is the region bounded by $x = 0, y = 0, y = 6, z = 4, z = x^2$.

[8+7]

