

I B. Tech I Semester Supplementary Examinations, May - 2017
MATHEMATICS-I
 (Com. to All Branches)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions
 All Questions carry **Equal** Marks

- ~~~~~
1. a) Solve $\frac{dy}{dx} = \frac{1}{(1+y^2)}(e^{\tan^{-1}x} - y)$. (7M)
 - b) A body is heated to 110°C and placed in air at 10°C . After 1 hour its temperature is 60°C . How much additional time is required for it to cool to 30°C ? (8M)
 2. a) Solve $(D^2 + 2)y = x^2e^{3x} + e^x \cos 2x$, where $D = \frac{d}{dx}$. (7M)
 - b) Solve $(D^2 + 3D + 2)y = e^{-x} + x^2 + \cos x$, where $D = \frac{d}{dx}$. (8M)
 3. a) If $u = x + y + z$, $uv = y + z$, $uvw = z$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (7M)
 - b) Examine the function $f(x, y) = \sin x + \sin y + \sin(x + y)$ for extreme. (8M)
 4. a) Trace the curve $x^3 + y^3 = 3axy$. (7M)
 - b) Trace the curve $r^2 = a^2 \cos 2\theta$. (8M)
 5. a) Find the length of the arc of parabola $y^2 = 4ax$ cut-off by latus rectum. (7M)
 - b) Find the surface area of the solid generated by the revolution of the asteroid $x = a \cos^3 t$, $y = a \sin^3 t$ about the y -axis. (8M)
 6. a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates. (5M)
 - b) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz$. (5M)
 - c) Change the order of integration and hence evaluate $I = \int_0^a \int_{\frac{x}{a}}^{\frac{\sqrt{x}}{a}} (x^2 + y^2) dx dy$. (5M)
 7. a) Evaluate divergence of $(2x^2z i - xy^2z j + 3yz^2 k)$ at the point $(1, 1, 1)$. (5M)
 - b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. (5M)
 - c) Evaluate $\text{Curl of } \vec{V} = yz i + 3zx j + z k$ at the point $(2, 3, 4)$. (5M)
 8. Verify Stoke's theorem for a vector field defined $\vec{F} = -y^3 i + x^3 j$, in the region $x^2 + y^2 \leq 1, z = 0$. (15M)