

**I B.Tech I Semester Supplementary Examinations, Aug. 2015
MATHEMATICS-I**(Common to Civil Engineering, Electrical & Electronics Engineering,
Mechanical Engineering, Electronics & Communication Engineering,
Computer Science & Engineering, Chemical Engineering, Electronics &
Instrumentation Engineering, Bio-Medical Engineering, Information
Technology, Electronics & Computer Engineering, Aeronautical
Engineering, Bio-Technology, Automobile Engineering, Mining and
Petroleum Technology)

Time: 3 hours

Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Solve $(3x^2y^2 + x^2) dx + (2x^3y + y^2) dy = 0$
(b) If the temperature of a body changes from $100^{\circ}C$ to $70^{\circ}C$ in 15 minutes, find when the temperature of the body will be $40^{\circ}C$, given that the temperature of the surroundings is $30^{\circ}C$ [7+8]
2. (a) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^x + e^{3x}$
(b) Solve $(D^2 + 16)y = 0$ [8+7]
3. (a) Find the maximum of $\frac{15xyz}{4x+2y+4z}$ given that $xyz = 8$.
(b) Find the maximum of x^3y^2z given that $x + 3y + 4z = 10$. [8+7]
4. (a) Trace the curve $y = (x - 2)(x + 3)(x - 4) ..$
(b) Trace the curve $r = \frac{1}{2} - \sin\theta$. [8+7]
5. (a) Find the surface of the solid generated by the revolution of cardioid $r = a(1 - \cos\theta)$ about the initial line.
(b) Find the surface of the solid generated by the revolution of the ellipse $x^2 + 4y^2 = 16$ about its Major axis. [8+7]
6. (a) Evaluate $\int \int r dr d\theta$ over the region bounded by the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$.
(b) Change the order of Integration & evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$ [8+7]
7. (a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 4$ and $z = x^2 + y^2 + 3$ at the point $(2, -1, 1)$.
(b) Prove that $\text{div grad } r^n = n(n+1)r^{n-2}$. [8+7]
8. (a) If s is surface of sphere with two units radius then show that $\int_S r \cdot N ds = 32\pi$
(b) Evaluate $\int_C f \cdot dr$ where $f = 3x^2 i + (2xz - y) j + z k$ along the straight line C from $(0, 0, 0)$ to $(2, 1, 3)$. [8+7]

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1. (a) Solve $(x^2y^3 + xy) \frac{dy}{dx} = 1$
(b) Find the orthogonal trajectory of the family of curves $r = a (1 + \sin \theta)$, where 'a' is a parameter [8+7]
2. (a) Solve $(D^2 - 3D + 2)y = e^x$
(b) Solve $(D^4 - a^4)y = 0$ [8+7]
3. (a) If $U = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, P.T. $x^2 \frac{\partial f}{\partial x} + y^2 \frac{\partial f}{\partial y} + z^2 \frac{\partial f}{\partial z} = 0$.
(b) Expand $u = x^y$ in powers of (x-1) and (y-1) up to third degree terms. [8+7]
4. (a) Trace the curve $r = a (1 + \cos 2\theta)$.
(b) Trace the curve $r = 3 + 2 \cos \theta$. [8+7]
5. (a) Find the length of the arc of the semi-cubical parabola $ay^2 = x^3$ from the vertex to the ordinate $x=5a$.
(b) Find the area of the surface of revolution generated by revolving one arc of the curve $y=\sin x$ about the x-axis. [8+7]
6. (a) Evaluate $\int \int \int_V dx dy dz$ where V is the finite region of space formed by the planes $x=0, y=0, z=0$ and $2x + 3y + 4z = 12$.
(b) Evaluate $\int \int_R y dx dy$ where R is the region bounded by the Parabolas $y^2 = 4x$ and $x^2 = 4y$. [8+7]
7. (a) Prove that $\nabla \times \{f(r)\vec{r}\} = 0$
(b) Find a unit vector which is perpendicular to the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point (1,2,5). [8+7]
8. (a) If $f = y \mathbf{i} + z \mathbf{j} + x \mathbf{k}$, find the circulation of f round the curve C, where c is the circle $x^2 + y^2 = 0, z = 0$.
(b) If $f = (x + y^2)\mathbf{i} - 2x\mathbf{j} + 2yz\mathbf{k}$, evaluate $\int_S f \cdot N ds$ where S is the surface of the plane $2x + y + 2z = 6$ in the first octant. [8+7]

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1. (a) Solve $\frac{dy}{dx} + y \sec x = \tan x$
 (b) Find the orthogonal trajectory of the family of curves $x^2 + y^2 = 2ax$, where 'a' is a parameter [8+7]
2. (a) Solve $y^{11} - 6y^1 + 25y = e^{2x} + x + \sin x$
 (b) Solve $y^{11} - 3y^{11} + 4y = 0, y(0) = 1, y^1(0) = -8, y^{11}(0) = -4$ [8+7]
3. (a) Show that $U = x^2 e^{-y} \cosh z, V = x^2 e^{-y} \sinh z, w = x^2 + y^2 + z^2 - xy - yz - zx$ are functionally dependent.
 (b) Determine whether the functions $U = \frac{x}{y-z}, V = \frac{y}{z-x}, W = \frac{z}{x-y}$ are dependent. If dependent find the relationship between them. [8+7]
4. (a) Trace the curve $x^2 + y^2 = xy$.
 (b) Trace the curve $y^2(2a - x) = x^3$. [8+7]
5. (a) Find the surface area generated by rotating the arc of the catenary $y = a \cosh \frac{x}{a}$ from $x=0$ to a about the x-axis.
 (b) Find the volume of the solid generated by revolving about the x-axis of the loop of the curve $y^2 = x^2 \frac{(a+x)}{a-x}$. [8+7]
6. (a) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz \, dx dy dz$.
 (b) Evaluate $\int \int \int (x+y+z) dx dy dz$ taken over the volume bounded by the planes $x = 0, x = 1; y = 0, y = 1; \text{ and } z = 0, z = 1$. [8+7]
7. (a) Find the unit normal vector to the level surfaces $x^2 y + 2xz = 4$ at the point (2,-2,3)
 (b) Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t$ at the point (1,1,1) [8+7]
8. Verify divergence theorem for $2x^2 y i - y^2 j + 4xz^2 k$ taken over the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$. [15]

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1. (a) Solve $(x^2 + y^3 + 6x) dx + (y^2 x) dy = 0$
 (b) If the population of a country doubles in 50 years, in how many years will it triple, assuming that the rate of increase is proportional to the number of inhabitants? [8+7]
2. (a) Solve $\frac{d^2y}{dx^2} + y = 0$ given that $y(0) = 2$, $y\left(\frac{\pi}{2}\right) = -2$
 (b) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 5e^{2x}$ [8+7]
3. (a) If $x = e^r \cos \theta$, $y = e^r \tan \theta$, show that $J\left(\frac{x,y}{r,\theta}\right) J\left(\frac{r,\theta}{x,y}\right) = 1$.
 (b) Find Taylor's series expansion of the $f(x,y) = \sin 2x$ about $x = \frac{\pi}{4}$. [8+7]
4. (a) Trace the curve $r = 4\theta$.
 (b) Trace the curve $r = \frac{1}{4} + 2 \sin \theta$. [8+7]
5. (a) Find the surface of the solid generated by revolution of the lemniscate $r^2 = a^2 \cos^2 \theta$ about the initial line.
 (b) Show that the whole length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$. [8+7]
6. (a) Evaluate $\int \int (x + y)^2 dx dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (b) Transform the following to Cartesian form and hence evaluate $\int_0^\pi \int_0^a r^3 \sin \theta dr d\theta$. [8+7]
7. (a) Prove that $\nabla \cdot \left\{ r \nabla \left(\frac{1}{r^3} \right) \right\} = \frac{3}{r^4}$.
 (b) Find the directional derivative of $\Phi(x,y,z) = x^2yz + 4xz^2$ at the point P = (1,-2,-1) in the directional of the normal to the surface $f(x,y,z) = x \log z - y^2$ at (-1,2,1). [8+7]
8. (a) If $f = y \mathbf{i} + z \mathbf{j} + x \mathbf{k}$, find the circulation of f round the curve C, where c is the circle $x^2 + y^2 = 0$, $z = 0$.
 (b) If $f = (x + y^2)\mathbf{i} - 2x\mathbf{j} + 2yz\mathbf{k}$, evaluate $\int_S f \cdot N ds$ where S is the surface of the plane $2x + y + 2z = 6$ in the first octant. [8+7]