

## I B. Tech I Semester Supplementary Examinations, November - 2020

MATHEMATICS-I  
(Com. to All branches)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions  
All Questions carry **Equal** Marks

1. a) Solve  $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ . (7M)  
b) Find the orthogonal trajectories of  $r = a(1 - \cos \theta)$ . (8M)
2. a) Solve  $(D^2 - p^2)y = \text{Sinh } px$ . (7M)  
b) Solve  $(D^2 - 4D + 4)y = x^2 \text{Sinx} + e^{2x} + 3$ . (8M)
3. a) Prove that  $u = \frac{x^2 - y^2}{x^2 + y^2}$ ,  $v = \frac{2xy}{x^2 + y^2}$  are functionally dependent and find the relation between them. (7M)  
b) Find the minimum value of  $x^2 + y^2 + z^2$  subject to the condition  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ . (8M)
4. a) Trace the curve  $r = 2\cos \theta + 1$ . (7M)  
b) Trace the curve  $y^2(a + x) = x^2(3a - x)$ . (8M)
5. a) Prove that the length of the arc of a loop of the curve  $9ay^2 = x(x - 3a)^2$  is  $4\sqrt{3}a$ . (7M)  
b) Find the area of the surface generated by revolving the loop of the curve  $9y^2 = x(x - 3)^2$  about the x-axis. (8M)
6. a) By changing the order of integration, evaluate  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2} dy dx$ . (7M)  
b) Evaluate  $\iiint_R (x + y + z) dz dy dx$  where R is the region bounded by the planes  $x = 0$ ,  $x = 1$ ,  $y = 1$ ,  $z = 0$ ,  $z = 1$ . (8M)
7. a) Find the angle between the normals to the surface  $2x^2 + 3y^2 = 5z$  at points  $(2, -2, 4)$  and  $(-1, -1, 1)$ . (7M)  
b) Prove that  $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$  (8M)
8. Verify Green's theorem in plane for  $\oint_c (x^2 - 2xy) dx + (x^2 y + 3) dy$  where c is the boundary of the region defined by  $y^2 = 8x$  and  $x = 2$ . (15M)