

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours**Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
 Answering the question in **Part-A** is Compulsory,
 Three Questions should be answered from **Part-B**

PART-A

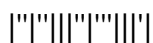
1. (i) Find the orthogonal trajectories of family of curves $r^n = a \sin n\theta$
- (ii) Solve $\frac{d^2y}{dx^2} - 4y = x \sinh x$
- (iii) Find the Laplace transform of $(\sqrt{t} - \frac{1}{\sqrt{t}})^3$
- (iv) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$, where $u = e^{x^2+y^2}$
- (v) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$
- (vi) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} = x^2 + y^2$

[3+3+4+4+4+4]

PART-B

- 2.(a) Solve $xy(1 + xy^2) \frac{dy}{dx} = 1$
- (b) Solve $y(xy + 2x^2y^3)dx + x(xy - x^2y^2)dy = 0$ [8+8]
- 3.(a) Solve $(D^2 - 1)y = x \sin 3x + \cos x$
- (b) A particle of mass m executes S.H.M in the line joining the points A and B, on a smooth table and is connected with these points by elastic strings whose tensions is equilibrium are each T . If l, l^1 be the extensions of the string beyond their natural lengths, find the time of oscillation. [8+8]
- 4.(a) Find the Laplace transform of $\frac{\cos at - \cos bt}{t} + t \sin at$.
- (b) Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x(\frac{\pi}{2}) = -1$ [8+8]
- 5.(a) Expand $e^x \log(1 + y)$ in powers of x and y up to terms of third degree.
- (b) In a plane triangle, find the maximum value of $\cos a \cos b \cos c$. [8+8]
- 6.(a) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$
- (b) Solve $q^2 = z^2 p^2 (1 - p^2)$. [8+8]
- 7.(a) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n\pi x}{l}$
- (b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = 0, u(l, t) = 0 \forall t$; $u(x, 0) = f(x)$ and $(\frac{\partial u}{\partial t})_{t=0} = g(x), 0 < x < l$.

[8+8]



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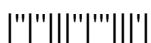
PART-A

- 1.(i) Find the orthogonal trajectories of the family of cardioids $r = a(1 + \cos\theta)$
- (ii) Solve the $(D^2 - 4D + 3)y = \sin 3x \cos 2x$
- (iii) Find the Laplace transform of $\sinh 3t \cos^2 t$
- (iv) If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{x}{y}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$
- (v) Solve $\frac{y^2 z}{x} p + xzq = y^2$
- (vi) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = e^{-x}$

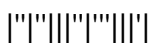
[3+3+4+4+4+4]

PART-B

- 2.(a) Solve $(y - xy^2)dx - (x + x^2y)dy = 0$
- (b) Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ [8+8]
- 3.(a) Solve $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \sin x$
- (b) An unchanged condenser of capacity C is charged by applying an e.m.f. $E \sin \frac{t}{\sqrt{LC}}$, through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on One of the plates is $\frac{EC}{2} \left\{ \sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right\}$ [8+8]
- 4.(a) Evaluate $L \left\{ t \int_0^t \frac{e^{-t}}{t} \sin t dt \right\}$
- (b) Find the inverse Laplace transform of $\log \left(\frac{s+1}{s-1} \right)$ [8+8]
- 5.(a) A rectangular box open at the top is to have volume of 32 cube ft. Find the dimensions of The box requiring least material for its construction.
- (b) Expand $f(x, y) = x^y$ in powers of (x-1) and (y-1) [8+8]



- 6.(a) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$
(b) Solve $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$ [8+8]
- 7.(a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$
(b) A tightly stretched string of length l , with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $\vartheta_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$. [8+8]



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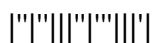
PART-A

- 1.(i) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$
- (ii) Solve $\frac{d^2y}{dx^2} + 2\frac{dx}{dy} + y = e^{2x} - \cos^2x$
- (iii) Find the Laplace transform of $e^{-1}\sin^2t$
- (iv) If $\sin u = \frac{x^2y^2}{x^2+y^2}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$
- (v) Solve $xp - yq = y^2 - x^2$
- (vi) Solve $4\frac{\partial^2z}{\partial x^2} - 4\frac{\partial^2z}{\partial x\partial y} + \frac{\partial^2z}{\partial y^2} = 16\log(x + 2y)$

[4+4+3+3+4+4]

PART-B

- 2.(a) Solve $\sec^2y\frac{dy}{dx} + xtany = x^3$
- (b) A body is originally at $80^\circ c$ cools down to $60^\circ c$ in 20 minutes, the temperature of the air being $40^\circ c$. What will be the temperature of the body after 40 minutes from the original. [8+8]
- 3.(a) Solve $(D^2 + 1)^2y = x^4 + 2\sin x \cos 3x$
- (b) Solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$. [8+8]
- 4.(a) Find the Laplace transform of $te^{-t}\sin 3t$.
- (b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ [8+8]
- 5.(a) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylors theorem.
- (b) Discuss the maxima and minima of $(x, y) = x^3y^2(1 - x - y)$. [8+8]
- 6.(a) Solve the partial differential equation $px+qy=1$
- (b) Solve $2z + p^2 + qy + 2y^2 = 0$ [8+8]
- 7.(a) Using the method of separation of variables, solve $py^3 + qx^2 = 0$
- (b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x, 0) = 3\sin n\pi x$, $u(0, t) = 0$ and $u(1, t) = 0$, where $0 < x < 1, t > 0$. [8+8]



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PART-A

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- (ii) Solve $(D^2 + 1)y = x^4 + 2\sin x \cos 3x$
- (iii) Find the inverse Laplace transform $\frac{s^2}{(s-2)^3}$
- (iv) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$, where $\log u = \frac{(x^3+y^3)}{(3x+4y)}$
- (v) Solve $(y^2 + z^2)p - xyq + zx = 0$
- (vi) Solve $\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^2 \partial y} + \frac{\partial^3 z}{\partial y^3} = e^{x+2y}$

[4+4+3+3+4+4]

PART-B

- 2.(a) Solve $(y - xy^2)dx - (x + x^2y)dy = 0$
- (b) Solve $y(xy + 2x^2y^3)dx + x(xy - x^2y^2)dy = 0$ [8+8]
- 3.(a) Solve $(D^2 + 1)^2y = x^4 + 2\sin x \cos 3x$
- (b) Solve $(D^4 + D^2 + 1)y = e^{-x/2} \cos \frac{\sqrt{3}}{2}x$. [8+8]
- 4.(a) Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$
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