

## I B. Tech I Semester Supplementary Examinations, November - 2020

## MATHEMATICS-I

(Com. to CE, EEE, ME, ECE, CSE, Chem E, EIE, IT, Auto E, Min E, Pet E, Agri E)

Time: 3 hours

Max. Marks: 75

**Answer any five Questions one Question from Each Unit**  
**All Questions Carry Equal Marks**

~~~~~

1. a) Test the convergence of the series  $\sum \frac{n!}{n^n}$  (8M)

b) Verify Cauchy's mean value theorem for  $f(x) = \sin x$ ,  $g(x) = \cos x$  in  $[0, \pi/2]$  (7M)

Or

2. a) Test the convergence of the series  $\sum \frac{(-1)^n}{n+1} x^{n+1}$  (8M)

b) Verify Rolle's theorem for  $f(x) = \frac{\sin x}{e^x}$  in  $[0, \pi]$  (7M)

3. a) Solve the ODE  $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$  (8M)

b) A bacterial culture, growing exponentially, increases from 100 to 400 gms in 10 hrs. How much was present after 3 hrs from the initial instant. (7M)

Or

4. a) Solve the ODE  $\tan y \frac{dy}{dx} + \tan x = \cos y \cdot \cos^3 x$  (8M)

b) If the temperature of the air is  $30^\circ\text{C}$  and substance cools from  $100^\circ\text{C}$  to  $70^\circ\text{C}$  in 15 minutes, find when the temperature will be  $40^\circ\text{C}$ . (7M)

5. a) Solve the DE  $(D^2 + 4)y = x \sin x$  (8M)

b) A particle of mass 4 gm executing simple harmonic motion has velocities cm/sec and 6 cm/sec respectively. When it is at distance 3 cm and 4 cm from the centre of its path. Find its period and amplitude. Find also the force acting on the particle when it is a distance 1cm from the centre. (7M)

Or

6. a) Solve the DE  $(D^2 + D)y = x^2 + 2x + 4$  (8M)

b) Solve the DE  $(D^2 - 2D)y = e^x \sin x$  by the method of variation of parameters (7M)

7. a) If  $x^2 = au + bv$ ,  $y^2 = au - bv$  prove that  $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$  (8M)

b) Evaluate the following using the relation  $JJ^1 = 1$  (7M)

if  $u = x + y + z$ ,  $u^2v = y + z$ ,  $u^3w = z$  then find  $J \left( \frac{u, v, w}{x, y, z} \right)$

Or

8. a) Find the points on the surface  $z^2 = xy + 1$  nearest to the origin. (8M)  
b) Expand  $f(x,y) = x^y$  in powers of  $(x - 1)(y - 1)$  using Taylor's series. (7M)
9. a) Evaluate  $\int_0^{\pi/2} \int_0^{4a \sin \theta} \frac{r}{\sqrt{16 - r^2}} dr d\theta$  (8M)  
b) Evaluate  $\iiint_V (x + y + z) dx dy dz$  taken over the volume bounded by  $x = 0, x = 1, y = 0, y = 1$  and  $z = 0, z = 1$ . (7M)

Or

10. a) Evaluate by change of order of integration  $\int_3^5 \int_0^{4/x} xy dx dy$  (8M)  
b) Evaluate  $\int_0^a \int_0^x \int_0^y x^3 y^2 z dx dy dz$ . (7M)