Code No: R201101 **R20** SET - 1

I B. Tech I Semester Regular Examinations, April- 2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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Unit - I

1. a) Examine the convergence of $\sum\left[\frac{1.4.7....(3n-2)}{3.6.9....3n}\right]^2$. (7M)

   b) If f(x) = log x and g(x) = x^2 in [a, b] with b > a > 1, using Cauchy's theorem prove that $\frac{\log b-\log a}{b-a}=\frac{a+b}{2c^2}$. (7M)

OR

2. a) Examine the convergence of $\frac{3}{5}-\frac{5}{7}+\frac{7}{10}-\frac{9}{13}+...$ (6M)

   b) Find Maclaurin's series expansion of the f(x) = $\sin^2 x$ about x=1. (8M)

Unit - II

3. a) Solve $\frac{dy}{dx}-2\frac{y}{x}-\frac{5x^2}{(x+2)(3-2x)}=0$. (7M)

   b) Suppose that an object is heated to 300^0F and allowed to cool in a room whose air temperature is 80^0F, if after 10 minutes the temperature of the object is 250^0F, what will be its temperature after 20 minutes. (7M)

OR

4. a) Find the orthogonal trajectories of $r^2=a\sin 2\theta$. (7M)

   b) Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (7M)

Unit - III

5. a) $Solve\ (D^2+3D+2)y=e^{-x}+cosx.$ (7M)

   b) Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$. (7M)

OR

6. a) In an L-C-R circuit, the charge q on a plate of a condenser is given by (7M)

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=Esinpt.$$

   The circuit is tuned to resonance so that q^2=1/LC. If initially the current ***I*** and the charge q be zero , find the current in the circuit.

   b) $Solve\ (D^2+4^2)y=tan2x\ ,by\ the\ method\ of\ Variation\ of\ parameters.$ (7M)
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Unit - IV

7. a) Determine whether the functions U = $\frac{x}{y-z}$, V = $\frac{y}{z-x}$, W= $\frac{z}{x-y}$ are dependent. If dependent find the relationship between them. (7M)

b) Expand the function $f(x, y) = xy^2 + \cos(xy)$ in powers of (x – 1) and $\left(y - \frac{\pi}{2}\right)$. (7M)

OR

8. a) Find the extreme values of the function $f(x, y) = xy(a - x - y)$. (8M)

b) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \log\left(\frac{x^2 + y^2}{xy}\right)$. (6M)

Unit - V

9. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2 y^2 (x + y) dy dx$. (7M)

b) Find the area bounded by the curve $= 2$, $4y = x^2$ and $y = 4$. (7M)

OR

10 a) By transforming into polar coordinates, evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, with b>a. (7M)

b) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2}\, dy dx$. (7M)

Code No: R201101 **R20** SET - 2

I B. Tech I Semester Regular Examinations, April - 2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

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All Questions Carry Equal Marks

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Unit - I

1\. a) Discuss the convergence of $\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{7.9}+...(x>0)$. (7M)

b) Find the region in which f(x) = $1 - 4x - x^2$ is increasing and the region in which it is decreasing using Mean Value Theorem. (7M)

OR

2\. a) Examine the convergence of $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...$ (7M)

b) Expand $\tan^{-1}x$ in powers of (x – 1) up to fourth degree term. (7M)

Unit - II

3\. a) Solve $\cosh x\frac{dy}{dx}+y\sinh x=2\cosh^2 x\sinh x.$ (7M)

b) A metal ball is heated to a temperature of 100^0C and at time t = 0 it is placed in water which is maintained at 40^0C. If the temperature of the ball reduces to 60^0C in 4 minutes, find the time at which the temperature of the ball is 50^0 C. (7M)

OR

4\. a) Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is the parameter. (7M)

b) Solve (xy sinxy + cos xy) ydx + (xy sinxy – cosxy) xdy = 0. (7M)

Unit - III

5\. a) $Solve\ (D^3-3D^2+4)y=e^{2x}+6+80cos2x.$ (7M)

b) Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ (7M)

OR

6\. a) The charge q(t) on the capacitor is given by the differential equation $10\frac{d^2q}{dt^2}+120\frac{dq}{dt}+1000q=17sin(2t).$ At initial time the current is zero and the charge on the capacitor is 0.0005 coulomb. Find the charge on the capacitor for t>0. (7M)

b) $Solve\ (D^2+4)y=Sec2x\ ,by\ the\ method\ of\ Variation\ of\ parameters.$ (7M)
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Unit - IV

7. a) Determine whether the functions $U = \frac{x}{y-z}$, $V = \frac{y}{z-x}$, $W = \frac{z}{x-y}$ are dependent. If dependent find the relationship between them. (7M)

b) Expand $f(x, y) = e^{x+y}$ in the neighborhood of (1, 1). (7M)

OR

8. a) Find the extreme values of the function $f(x, y) = x^2 y + y^2 + x^4$. (7M)

b) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$. (7M)

Unit - V

9. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dydx$. (7M)

b) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. (7M)

OR

10 a) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\, dxdy$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2}\, dydx$ by changing into polar coordinates. (7M)

Code No: R201101 **R20** SET - 3

I B. Tech I Semester Regular Examinations, April - 2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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Unit - I

1. a) Examine the convergence $\sum \frac{1}{(n^{3/2}+n+1)}$. (7M)

   b) Verify Lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in [0,4]. (7M)

OR

2. a) Test for convergence of $1 - \frac{x^2}{2!} + \frac{x^4}{4} - \frac{x^6}{6!} + \ldots (0 < x < 1)$. (7M)

   b) Find Taylor's series expansion of the f(x) = $\cos x$ about x = $\frac{\pi}{3}$. (7M)

Unit - II

3. a) Solve $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1} x$. (7M)

   b) If the air is maintained at 30^0C and the temperature of the body cools from 80^0C to 60^0C in 12 minutes, find the temperature of the body after 24 minutes. (7M)

OR

4. a) Find the orthogonal trajectories of $r = a(1-\cos\theta)$. (7M)

   b) Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$. (7M)

Unit - III

5. a) $Solve\ (D^2 - 3D + 2)y = 2x^2$. (7M)

   b) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$. (7M)

OR

6. a) $Solve\ (D^2 - 3D + 2)y = 2x^2$. (7M)

   b) $Solve\ (D^2 + 1)y = cosec\ x\ by\ the\ method\ of\ Variation\ of\ parameters$. (7M)

Unit - IV

7. a) Check whether u = $\frac{x^2 - y^2}{x^2 + y^2}$, v = $\frac{2xy}{x^2 + y^2}$ are functionally dependent and find the relation between them. (7M)
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b) Expand $e^x \cos y$ by Taylor's theorem about the point $\left(1, \frac{\pi}{4}\right)$ up to the second degree terms. (7M)

OR

8. a) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 + 72x - 15y^2$. (7M)

b) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$. (7M)

Unit - V

9. a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx.}{1+x^2+y^2}$. (7M)

b) Find the area bounded by pair of curve $y = 2 - x$ *and* $y^2 = 2(2 - x)$. (7M)

OR

10 a) By changing the order of integration, evaluate $\int_0^1 \int_1^{2-x} xydxdy$. (7M)

b) Using spherical polar coordinates, evaluate $\iiint xyz\, dxdydz$ taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. (7M)

I B. Tech I Semester Regular Examinations, April - 2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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Unit - I

1. a) Test for convergence of $1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+...$ (7M)

   b) Prove using Mean Value Theorem $|\sin u - \sin v| \le |u-v|$. (7M)

OR

2. a) Examine the convergence of $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...\quad (x>0)$ (7M)

   b) Find the MacLaurin's expansion of $\log(1+e^x)=\log 2+\frac{x}{2}+\frac{x^2}{8}-\frac{x^4}{192}+....$ (7M)

Unit - II

3. a) Solve $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$. (7M)

   b) In 20 minutes, a body changes its cools from 80^0C to 60^0C, and the temperature of air being 40^0C. What will be the temperature of the body after 40 minutes from the original? (7M)

OR

4. a) Find the orthogonal trajectories of the family of curves: $r^n = a^n \sin n\theta$. (7M)

   b) Solve $(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$. (7M)

Unit - III

5. a) $Solve\ (D^3-4D^2-D+4)y=e^{3x}cos2x.$ (7M)

   b) Solve $(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}+y=\sin(2\log(1+x))$ (7M)

OR

6. a) $Solve\ (D^2+D)y=x^2+2x+4.$ (7M)

   b) $Solve\ (D^2-2D+1)y=\ e^x logx\ , by\ method\ of\ variation\ of\ parameters.$ (7M)

Unit - IV

7. a) Check whether u = $x^2e^{-y}\cosh z$, v = $x^2e^{-y}\sinh z$, w = $x^2+y^2+z^2-xy-yz-zx$ are functionally dependent. If dependent find the relationship between them. (7M)
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b) Expand the function $f(x,y)=\tan^{-1}(xy)$ in powers of (x – 1) and (y + 1). (7M)

OR

8. a) Find the minimum distance from the point (1, 2, 0) on to the cone $z^2 = x^2 + y^2$. (7M)

b) Find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ using Euler's theorem for the function $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. (7M)

Unit - V

9. a) Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y} dxdydz.$ (7M)

b) Find the area lying between the circle $x^2+y^2=a^2$ and the plane $x+y=a$ in the first quadrant. (7M)

OR

10 a) By changing the order of integration, evaluate $\int_0^3\int_1^{\sqrt{4-y}}(x+y)dxdy.$ (7M)

b) Using spherical polar coordinates , evaluate $\iiint \frac{xyz\ dxdydz}{\sqrt{x^2+y^2+z^2}}$ taken over the volume (7M)

Bounded by the sphere $x^2+y^2+z^2=a^2$ in the first octant.