

I B. Tech I Semester Regular Examinations, July/August-2021

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours

Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

1 a) Examine the convergence of $\sum \frac{[(n+1)!]^2 x^{n-1}}{n}$, ($x > 0$) (7M)

b) Find Maclaurin's series expansion of the $f(x, y) = \sin^2 x$ and hence find the approximate value of $\sin^2 16^\circ$. (7M)

Or

2. a) Prove using mean value theorem $|\sin u - \sin v| \leq |u - v|$. (7M)

b) Examine the convergence of $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ ($x > 0$). (7M)

3. a) Solve $(x + 2y^3) \frac{dy}{dx} = y$. (7M)

b) Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ (7M)

Or

4. a) Find the orthogonal trajectories of $r^2 = a \sin 2\theta$. (7M)

b) Solve $(xysinx + cosxy) ydx + (xysinxy - cosxy) xdy = 0$. (7M)

5. a) Solve $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$ (7M)

b) In an L-C-R circuit, the charge q on a plate of a condenser is given by (7M)

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt$$

The circuit is tuned to resonance so that $q^2 = 1/LC$. If initially the current I and the charge q be zero, show that, for small values of R/L , the current in the circuit at time t is given by $(Et/2L)\sin pt$.

Or

6. a) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters. (7M)

b) Solve $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$. (7M)

7. a) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (7M)

b) Investigate the maxima and minima, if any, of the function $f(x) = x^3 y^2 (1 - x - y)$. (7M)

Or

8. a) Prove that $u = \frac{x^2 - y^2}{x^2 + y^2}$, $v = \frac{2xy}{x^2 + y^2}$ are functionally dependent and find the relation between them. (7M)

b) Expand $f(x, y) = e^{x+y}$ in the neighborhood of (1, 1). (7M)

9. a) Evaluate $\iint_R xy dx dy$ where R is the region bounded by the x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. (7M)

b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$. (7M)

Or

10 a) Evaluate the following integral $\int_0^{\pi/2} \int_0^{a \sin \theta} \int_0^{(a^2 - r^2)/a} r dr d\theta dz$ (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{x^2 + y^2} dy dx$ by changing into polar coordinates. (7M)