B.Tech I Year (R13) Regular & Supplementary Examinations May/June 2015 MATHEMATICS - II

(Common to EEE, ECE, EIE, CSE & IT)

Time: 3 hours

3

Max. Marks: 70

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## Part – A

(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
  - (a) Define rank of a matrix.
  - (b) Define a Skew-Hermitian matrix with example.
  - (c) Develop an algorithm using Newton-Raphson method, to find the square root of a positive number N.
  - (d) Write Newton-Gregory forward and backward interpolation formula.
  - (e) Write Milne's predictor-Corrector formulae to solve the ODE  $\frac{dy}{dx} = f(x, y)$  with  $y(x_0) = y_0$ .
  - (f) Write Dirichlet conditions for the expansion of f(x) in Fourier series.
  - (g) Define finite Fourier sine and cosine transforms and their inversion formulae in 0 < x < L.
  - (h) State initial and final value theorems for Z-transform.
  - (i) Derive the partial differential equation by eliminating the constants a and b from the equation  $z = a x^2 + by^2$ .
  - (j) Form partial differential equation by eliminating the arbitrary functions from  $z = f(x) + e^x g(x)$ .

(Answer all five units, 5 X 10 = 50 Marks)

2 (a) Determine the values of  $\lambda$  for which the following system of equations has non-trivial solutions. Find them:

 $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$  $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$  $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0.$ 

(b) Prove that the Eigen values of Hermitian matrix are real.

Find the Eigen vectors of the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$  and hence reduce the quadratic form

 $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2 x_3$  to a 'sum of squares'. Also write its nature.

- 4 (a) Determine the root of  $x e^x 2 = 0$  by method of false position.
  - (b) Using Lagrange's formula, express the function  $\frac{x^2+6x-1}{(x^2-1)(x-4)(x-6)}$  as a sum of partial fractions.
- OR 5 (a) Fit a least squares quadratic curve  $y = a_0 + a_1x + a_2x^2$  to the following data  $x \quad 1 \quad 2 \quad 3 \quad 4$   $y \quad 1 \quad 7 \quad 1 \quad 8 \quad 2 \quad 3 \quad 3 \quad 2$ Estimate y(2.4). WWW Manaresults Co.in

(b) Evaluate  $\int_{0}^{0.6} e^{-x^2} dx$  by using Simpson's  $\frac{1}{3}$  rule taking seven ordinates.

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UNIT - III

- 6 (a) Obtain Picard's second approximate solution of the initial value problem  $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ , y(0) = 0. Find y(1).
  - (b) Given that  $\frac{dy}{dx} = 2 + \sqrt{xy}$ , y(1) = 1. Find y(2) in steps of 0.2 using the Euler's method. OR
- 7 Find the two half –range expansions of the function:

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$
UNIT - IV

- 8 (a) Express the function  $f(x) = \begin{cases} 1 & for |x| \le 1 \\ 0 & for |x| \ge 1 \end{cases}$  as a Fourier integral. Hence evaluate  $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda} d\lambda$ .
  - (b) Find: (i)  $Z\{\cos\theta + i \sin\theta\}^n$ . (ii)  $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$  by using convolution theorem. OR
- 9 (a) Find the Fourier transform of  $e^{-a^2 x^2}$ , a > 0. Hence deduce that  $e^{-\frac{x^2}{2}}$  is self reciprocal in respect of Fourier transform.
  - (b) Using the Z-transform solve the difference equation:  $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$  with  $u_0 = 0, u_1 = 1$ .

10 Solve the wave equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$  under the conditions

$$y(0,t) = 0, y(L,t) = 0 \text{ for all } t; \ y(x,0) = f(x) \text{ and } \left(\frac{\partial y}{\partial t}\right)_{t=0} = g(x), 0 < x < L.$$

11 A bar AB of length 10 cm has its ends A and B kept at  $30^{\circ}$  and  $100^{\circ}$  temperatures respectively, until steady-state condition is reached. Then the temperature at A is lowered to  $20^{\circ}$  and that at B to  $40^{\circ}$  and these temperatures are maintained. Find the subsequent temperature distribution in the bar.

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