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BOARD DIPLOMA EXAMINATION, (C–14)

MARCH/APRIL—2017

DCE—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours]　　　　　　　　　　　　　　[*Total Marks* : 80

PART—A　　　　　　　　　　3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answer should be brief and straight to the point and shall not exceed *five* simple sentences.

1. Resolve $\dfrac{2x\ 1}{(x\ 1)(2x\ 3)}$ into partial fractions.

2. If $A\ B\ \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ and $A\ B\ \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$, then determine A.

3. If　is a cube root of unity, prove that

$$\begin{vmatrix} 1 & & ^2 \\ ^2 & & \\ & ^2 & 1 \end{vmatrix}\ 0$$

4. If $A\ B\ 45$, prove that $(1\ \tan A)(1\ \tan B)\ 2$.

5. Prove that $\cos^4 A - \sin^4 A = \cos 2A$.

6. Find the modulus of $\dfrac{9}{2-i\sqrt{5}}$.

7. Find the equation of the straight line passing through the points $(12, -1)$ and $(13, -4)$.

8. Find the equation of the point circle with centre $(7, -9)$.

9. Evaluate $\displaystyle \lim_{x \to 0} \dfrac{\tan 37x}{\sin 11x}$.

10. Find $\dfrac{dy}{dx}$, if $y = x^6 - 6x^5 + 3x^2 - 1$.

PART—B   10×5=50

Instructions : (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

(3) Answers should be comprehensive and the criterion for valuation is the content but not the length of the answer.

11. *(a)* Show that

$$\begin{vmatrix} a-b-2c & a & b \\ c & b-c-2a & b \\ c & a & c-a-2b \end{vmatrix} = 2(a-b-c)^3$$

(b) Solve the following system of equations by using Cramer's rule :
$$x - 2y + z = 3$$
$$3x - y + z = 4$$
$$x - y + 2z = 6$$

12. *(a)* $\cos 10 \cdot \cos 30 \cdot \cos 50 \cdot \cos 70 = \dfrac{3}{16}$

(b) Solve $\tan^{-1}(1-x) + \tan^{-1}(1+x) = \tan^{-1}\dfrac{1}{2}$.

13. *(a)* Solve $\cos\theta = \sqrt{3}\sin\theta$.

(b) Solve the triangle ABC, if $a = 13$, $b = 14$ and $c = 15$.

14. *(a)* Find the equation of the ellipse whose axes are the coordinate axes and whose foci are $(\pm 5, 0)$ and $e = \dfrac{1}{5}$.

(b) Find the equation of the rectangular hyperbola whose focus is $(3, 4)$ and directrix is $4x - 3y + 1 = 0$.

15. *(a)* Differentiate $y = x^{\log x}$ w.r.t. x.

(b) Find $\dfrac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

16. *(a)* Differentiate $\sin^{-1}\dfrac{2x}{1+x^2}$ w.r.t. $\cos^{-1}\dfrac{1-x^2}{1+x^2}$.

(b) If $u = \log(x + y + z)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 1$.

17. *(a)* Find the length of tangent, normal, sub-tangent and sub-normal to the curve $x^2 + y^2 - 6x + 2y + 5 = 0$, at $(2, -1)$.

(b) A light is hung 8 m, directly above a straight horizontal floor. A man of 2 m tall is walking away from the lamp at the rate of 5·4 m/min. Find the rate at which his shadow is lengthening.

18. *(a)* Find the dimensions of a rectangle of maximum area having a perimeter of 36 ft.

(b) An error of 0·05 cm is committed in measuring a length of 10 cm. If so, find the absolute error, relative error and percentage error.

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