6017

BOARD DIPLOMA EXAMINATION, (C–16)
MARCH/APRIL—2018
DCE—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours] [*Total Marks* : 80

PART—A 3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answers should be brief and straight to the point and shall not exceed *five* simple steps.

1. Resolve $\dfrac{2x+3}{(x+1)(x+2)}$ into partial fractions.

2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X = A + B$, then find X.

3. If $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$, then find $(A+B)^T$.

4. If $A + B = 45º$, then prove that $(1+\tan A)(1+\tan B) = 2$.

5. Prove that $\dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$.

6. Find the modulus of $\dfrac{7+24i}{3+4i}$.

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7. Find the perpendicular distance of a point $(3, -5)$ from the line $3x - 4y - 26 = 0$.

8. Find the equation of a straight line parallel to $x - 2y + 1 = 0$ and passing through the point $(-1, 2)$.

9. Evaluate $\lim\limits_{x \to 0} \dfrac{\tan 3x}{\sin 5x}$.

10. If $y = \log\left(\dfrac{1 - x^2}{1 + x^2}\right)$, then find $\dfrac{dy}{dx}$.

PART—B   10×5=50

Instructions : (1) Answer *any* **five** questions.

                 (2) Each question carries **ten** marks.

                 (3) Answers should be comprehensive and the criterion for valuation is the content but not the length of the answer.

11. *(a)* Solve the equation $2x + 3y + z = 1$, $x + 4y + 2z = 3$ and $4x + y + 3z = 11$ by using Cramer's method.

    (b) Show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$$

12. *(a)* If $\sin x + \sin y = \dfrac{3}{4}$ and $\sin x - \sin y = \dfrac{2}{5}$, then prove that

$$8 \cot \frac{x + y}{2} = 15 \cot \frac{x - y}{2}$$

    (b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that

$$x + y + z = xyz$$

13. *(a)* Solve $2 \sin^2 \theta + 3 \cos \theta + 3 = 0$.

    (b) In triangle ABC, if $\dfrac{1}{a + c} + \dfrac{1}{b + c} = \dfrac{3}{a + b + c}$, show that $C = 60°$.

14. *(a)* Find the equation of the circle passing through the points $(0, 0)$, $(1, -2)$ and $(2, 0)$.

(b) Find the eccentricity, foci, length of latus rectum and equation of directrices of the ellipse $16x^2 + 9y^2 = 144$.

15. *(a)* If $y = x^{x^{x \cdots}}$, then prove that $\dfrac{dy}{dx} = \dfrac{y^2}{x(1 - y \log x)}$.

(b) Differentiate $e^{\tan^{-1} x}$ with respect to $\tan^{-1} x$.

16. *(a)* If $y = \sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$.

(b) If $z = \log(e^x + e^y)$, then prove that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 1$.

17. *(a)* Find the equations of tangent and normal to the curve $Y = x^2 - 6x + 11$ at $(6, 11)$.

(b) The radius of a sphere is decreasing at the rate of $0{\cdot}2$ cm/sec. Find the rate at which its volume is decreasing when the radius of the sphere is 10 cm.

18. *(a)* The sum of two numbers is 24. Find the numbers when the sum of their squares is minimum.

(b) If an error of 2% is made in measuring the side of a square plate, find % error in its area.

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