

7017

BOARD DIPLOMA EXAMINATION, (C-20)

MAY—2023

DCE - FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 Hours ]

[ Total Marks : 80

## PART—A

3×10=30

- Instructions :** (1) Answer **all** questions.  
 (2) Each question carries **three** marks.

1. If  $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$  and  $f : A \rightarrow B$  is a function such that  $f(x) = \cos x$ , then find the range of  $f$ .

2. Resolve  $\frac{x-1}{(x-2)(x+3)}$  into partial fractions.

3. If  $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ , then find  $3B - 2A$ .

4. If  $A + B = \frac{\pi}{4}$ , then prove that  $(1 + \tan A)(1 + \tan B) = 2$ .

5. Prove that  $\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$ .

6. Find the modulus of the complex number  $\frac{7+24i}{3-4i}$ .

7. Find the equation of the straight line passing through (3, -4) and parallel to the line  $x + 7y + 1 = 0$ .
8. Evaluate  $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8}$
9. Find the derivative of  $3 \cos x + \log x + 21x + 5$ .
10. Find the derivative of  $e^{\sin^{-1} x}$ .

**PART—B**

8×5=40

- Instructions :** (1) Answer **all** questions.  
 (2) Each question carries **eight** marks.

11. (a) Find the value of  $x$ , if the matrix  $\begin{bmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{bmatrix}$  is singular.

**(OR)**

- (b) Solve the following system of equations using Cramer's rule  
 $2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$ .

12. (a) Prove that  $\frac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A + B), A \neq B$ .

**(OR)**

- (b) Prove that  $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$ .

13. (a) Solve  $\cos \theta + \sin \theta = \sqrt{2}$ .

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(OR)

(b) In any  $\triangle ABC$ , if  $\angle A = 60^\circ$ , then prove that  $\frac{b}{c+a} + \frac{c}{a+b} = 1$ .

14. (a) Find the equation of the circle passing through the points (0, 0), (6, 0) and (8, 4).

(OR)

(b) Find the equation of the rectangular hyperbola whose focus is the point (-1, -3), and directrix is the line  $2x + y + 1 = 0$ .

15. (a) If  $y = x^{x^{x^{\dots}}}$ , then show that  $\frac{dy}{dx} = \frac{y^2}{x(1 - y \log x)}$ .

(OR)

(b) If  $u(x, y) = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ , then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ .

### PART—C

10×1=10

- Instructions :** (1) Answer the following question.  
(2) The question carries **ten** marks.

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16. Find the lengths of tangent, normal, sub tangent and sub normal to the parabola  $y^2 = 4x$  at (1, 2).

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