

Code No: 111AB

R13

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, May/June - 2017

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, BME, IT, MCT, ETM, MMT, AE, AME, MIE, PTM, AGE)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

Part- A (25 Marks)

- 1.a) Find the inverse of the matrix $\text{diag}[a, b, c]$, $a \neq 0, b \neq 0, c \neq 0$. [2]
- b) Find the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -2 \\ 2 & -2 & 3 \end{bmatrix}$. [3]
- c) Find C of the mean value theorem, if $f(x) = x(x-1)(x-2)$ and $a = 0, b = 0.5$. [2]
- d) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$ find $\frac{\partial(u,v)}{\partial(x,y)}$. [3]
- e) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [2]
- f) Evaluate $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx dy dz$. [3]
- g) Find the general solution of $(4D^2 + 4D + 1)y = 0$. [2]
- h) Find $\frac{1}{D^2 - 1} e^x$. [3]
- i) Find $L[te^{2t}]$. [2]
- j) Find $L^{-1}\left\{\frac{s}{(s-1)(s-2)}\right\}$. [3]

Part-B (50 Marks)

2. Find for what values of λ the equations $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$ have a solution and solve them in each case. [10]

OR

3. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify Cayley-Hamilton theorem. Find A^{-1} using Cayley – Hamilton theorem. [10]

- 4.a) Verify Lagrange's mean value theorem for the function $f(x) = e^x$ in $[0, 1]$.

- b) Expand $\log \cos(x + h)$ in powers of h by Taylor's theorem. [5+5]

OR

- 5.a) If u and v are functions of x and y and $J = \frac{\partial(u, v)}{\partial(x, y)}, J' = \frac{\partial(x, y)}{\partial(u, v)}$ then prove that $JJ' = 1$.

- b) Find the minimum values of $x^2 + y^2 + z^2$ if $x + y + z = 3a$. [5+5]

6. Show that $\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$. [10]

OR

7. Change the order of integration and evaluate $\int_0^a \int_{\frac{y}{a}}^{\sqrt{y/a}} (x^2 + y^2) dy dx$. [10]

- 8.a) Solve $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$.

- b) If the surroundings are maintained at 30°C and the temperature of body cools from 80°C to 60°C in 12 minutes, find the temperature of body after 24 minutes. [5+5]

OR

- 9.a) Solve $(D^2 + 3D + 2)y = e^{-x} + \cos x$.

- b) Solve $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$. [5+5]

- 10.a) Find $L^{-1} \left\{ \log \left(1 + \frac{a^2}{s^2} \right) \right\}$

- b) Find $L^{-1} \left\{ \frac{s}{(s^2 + 4)^2} \right\}$ by convolution theorem. [5+5]

OR

11. Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1$ by Laplace transform. [10]