Code No: 111AB **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, November/December - 2015

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A **(25 Marks)**

1.a) Define Elementary matrix with an example. [2]

b) Prove that an orthogonal set of vectors is linearly independent. [3]

c) Check whether the functions $u = e^x \sin y, v = e^x \cos y$ are functional dependent or not. If so find the relation between them. [2]

d) Find the stationary points of $u(x, y) = \sin x \sin y \sin(x + y)$ where $0 < x, y < \pi$. [3]

e) Evaluate $\int_0^\infty a^{-bx^2} dx$. [2]

f) Evaluate $\iint r \sin\theta \, dr \, d\theta$ over the cardioid $r = a(1 - \cos\theta)$ above the initial line. [3]

g) Solve $(y + x)dx = (y - x)dx$. [2]

h) Find Particular Integral of $(D^6 - D^4)y = x^2$. [3]

i) Define Unit impulse function. [2]

j) State and prove linear property of Laplace transforms. [3]

PART-B **(50 Marks)**

2.a) Reduce the quadratic form $2x_1x_2 + 2x_1x_3 + 2x_2x_3$ to canonical form.

b) Determine the values of a, b, c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal. [5+5]

OR

3.a) If A is an n x n matrix and $A^2 = A$, then show that each Eigen value of A is 0 or 1.

b) For what values of λ, the system of equations $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case. [5+5]

4.a) Prove that $u = x + y + z, v = xy + yz + zx, w = x^2 + y^2 + z^2$ are functional dependent and find the relation between them.

b) If $x = u(1 - v); y = uv$ prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$. [5+5]

OR

5.a) State and verify Rolle's theorem for the function $f(x) = x^{2m-1}(a - x)^{2n}$ in $(0, a)$.

b) Show that $h < e^h - 1 < he^h$ for $h \neq 0$. [5+5]

6.a) Evaluate $\iint (x^2+y^2)dxdy$ over the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant by using the transformation $x=au$ and $y=bv$.

b) Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$. [5+5]

OR

7.a) Evaluate $\iint x^{m-1}y^{n-1}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

b) Evaluate $\int_0^\infty \frac{xdx}{\left(1+x^6\right)}$ using Γ- β functions. [5+5]

8. Radium decomposes at a rate proportional to the quantity of radium present. Suppose it is found that in 25 years approximately 1.1% of a certain quantity of radium has decomposed. Determine approximately how long will it take for one-half of the original amount of radium to decompose. [10]

OR

9.a) Solve $xdx+ydy=\frac{xdy-ydx}{x^2+y^2}$.

b) Solve by the method of variation of parameters $(D^2-2D)y=e^x\sin x$. [5+5]

10.a) Find the Laplace transform of the function $f(t)=\begin{cases} t & 0<t<a \\ -t+2a & a<t<2a \end{cases}$

b) Find inverse Laplace transform of the function $\frac{1}{s^2\left(s+3\right)}$. [5+5]

OR

11.a) Using Laplace transform, solve $(D^2+1)x=t\cos 2t$ given $x=0, \frac{dx}{dt}=0$ at $t=0$.

b) Using Convolution theorem, evaluate $L^{-1}\left\{\frac{1}{s\left(s^2+2s+2\right)}\right\}$. [5+5]

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