Code No: 111AB **R13**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year Examinations, October/November - 2016
MATHEMATICS-I
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A **(25 Marks)**

1.a) Define Eigen vector of a matrix. [2]
b) Write the working procedure to solve the system of non-homogenous equations. [3]
c) Verify for $x = u,\ y = u\tan v,\ z = w,\ J\left(\frac{x, y, z}{u, v, w}\right) \times J'\left(\frac{u, v, w}{x, y, z}\right) = 1.$ [2]
d) Give an example of a function that is continuous on [-1, 1] and for which mean value theorem does not hold, explain. [3]
e) Show that $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1).$ [2]
f) Evaluate $\int_0^1 \int_1^{2-x} xydxdy.$ [3]
g) Explain the method of solving Bernoulli equation. [2]
h) Solve $(D^4 + 2D^2n^2 + n^4)y = 0.$ [3]
i) State and prove change of scale property of Laplace transforms. [2]
j) Prove that $L^{(-1)}\{F(s)\} = f(t)$ and $f(0) = 0$ then $L^{(-1)}\{sF(s)\} = \frac{df}{dt}.$ [3]

PART-B **(50 Marks)**

2. Determine a non-singular matrix P such that P^TAP is a diagonal matrix, where

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$$ [10]

OR

3.a) Show that the two matrices A, $C^{-1}AC$ have the same latent roots.
b) For a matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$. [5+5]

4.a) Find the minimum and maximum values of $\sin x + \sin y + \sin(x+y).$
b) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, x^2 + y^2 + z^2 \neq 0$ then evaluate $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$ [5+5]

OR

5.a) Prove that $\frac{\pi}{6}+\frac{1}{5\sqrt{3}}<\sin^{-1}\left(\frac{3}{5}\right)<\frac{\pi}{6}+\frac{1}{8}$.

b) Verify Lagrange's mean value theorem for $f(x)=\begin{cases} x\sin\frac{1}{x} & (x\neq 0) \\ 0 & (x=0)\end{cases}$ in [-1. 1]. [5+5]

6.a) Evaluate $\iiint_V x^{l-1}y^{m-1}z^{n-1}dxdydz$ where V is the region $x\geq 0, y\geq 0, z\geq 0$ and the plane $x+y+z<1$.

b) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx\,(c>1)$ in terms of Gamma function. [5+5]

OR

7.a) By changing the order of integration and evaluate $\int_0^b\int_0^{\frac{a\sqrt{b^2-y^2}}{b}} xydydx$.

b) Find the area enclosed by the parabolas $x^2=y$ and $y^2=x$. [5+5]

8.a) The number N of bacteria in a culture grows at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hour?

b) Solve $(x-y)dx-dy=0, \quad y(0)=2$. [5+5]

OR

9. Solve $(D^2\text{-}4D+4)y=x^2\sin x+e^{2x}+3$. [10]

10.a) State and prove convolution theorem for Laplace transforms.

b) Find the Laplace transform of $f(t)=|t-1|+|t+1|, t\geq 0$. [5+5]

OR

11.a) Solve the differential equation using Laplace transforms

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=e^{-t};\ x(0)=0, x'(0)=1.$$

b) Evaluate $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$. [5+5]

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