

Code No: 113AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, February/March - 2016

MATHEMATICS – III

(Common to EEE, ECE, EIE, ETM, AGE)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A**(25 Marks)**

- 1.a) Prove that $x=2$ is a regular singular point for the differential equation $x(2-x)y'' - 2(x-1)y' + 2y = 0$ [2]
- b) Find the particular integral of $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 8y = x^3$ [3]
- c) Evaluate $\int_{-1}^1 P_0(x) dx$ [2]
- d) Prove that $J_0(0) = 1$ [3]
- e) Find the value of a if $\cos ax \sin hy$ is harmonic. [2]
- f) Find the points at which $f(z) = \frac{z}{(z^2 - z)}$ is not analytic. [3]
- g) Find the residue of $\frac{2z+3}{z^2 - z - 2}$ at $z = -1$ [2]
- h) Expand $z \cos \frac{1}{z}$ [3]
- i) The fixed points of $f(z)$ are the points where $f(z) = z$ [2]
- j) Find the critical points of $w = \sin z$ [3]

PART-B**(50 Marks)**

2. Solve the differential equation in series. $2x \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - y = 0$ around $x = 0$ [10]

OR

3. Solve the differential equation in series. $\frac{d^2y}{dx^2} + xy = 0$ [10]

- 4.a) Prove that $J_{n-1} = \frac{2}{x}(nJ_n - (n+2)J_{n+2} + (n+4)J_{n+4} + \dots)$

- b) Express $x^2 - 3x + 4$ in terms of Legendre Polynomials [5+5]

OR

- 5.a) Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

- b) Express $x^2 - 4x + 7$ in terms of Legendre Polynomials. [5+5]

- 6.a) Find an analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$
- b) Evaluate the integral $\int_C \frac{\sin^2 z}{(z - \frac{\pi}{6})^3} dz$ where $C : |z| = 1$ [5+5]

OR

- 7.a) Find the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$
- b) Evaluate $\int_C \frac{(z^3 - z) dz}{(z - 2)^3}$ where C is $|z| = 3$ [5+5]
8. Expand $\frac{1}{z^2 - 3z + 2}$
- a) $|z| > 2$ b) $1 < |z| < 2$ [5+5]

OR

9. Find the residue at the singular points of the function $\frac{z^2}{(z - 1)^2(z + 2)}$. [10]
- 10.a) Find the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$
- b) Find the bilinear mapping which maps the points $z = 1, i, -1$ into $0, 1, \infty$. [5+5]

OR

- 11.a) Find the image of the infinite stripe $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the mapping $w = \frac{1}{z}$
- b) Find the image of $|z| < 1$ and $|z| > 1$ under the transformation $W = \frac{iz + 1}{z + i}$ [5+5]

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