R13

Code No: 113AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, March - 2017
MATHEMATICS – III
(Common to AGE, ECE, EEE, EIE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

1.a) Solve the differential equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+3y=0$. [2]

b) Define Regular singular point of a differential equation with example. [3]

c) Write Rodrigue's formula and Generating function of Legendre's polynomials. [2]

d) Show that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$. [3]

e) Show that the function $f(z)=e^z$ is entire. [2]

f) Evaluate, using Cauchy's integral formula $\int_C \frac{\log z}{(z-1)^3}dz$, where C is $|z-1|=\frac{1}{2}$. [3]

g) Determine the poles of the function $f(z)=\frac{z+1}{z^2(z-2)}$ and the residue at each pole. [2]

h) Find the Laurent series of $f(z)=\frac{(z-2)(z+2)}{(z+1)(z+4)}$, for $|z|>4$. [3]

i) Find the invariant points of the transformation $w=(z-i)^2$. [2]

j) Determine the image of the region $|z-3|=5$ under the transformation $w=\frac{1}{z}$. [3]

PART-B

(50 Marks)

2.a) Solve the differential equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=x^2\sin(\log x)$.

b) Apply the Frobenius method to solve ODE $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$. [5+5]

OR

3.a) Solve the differential equation $(2x+5)^2\frac{d^2y}{dx^2}-6(2x+5)\frac{dy}{dx}+8y=6x$.

b) Apply the power series method to solve ODE $\frac{d^2y}{dx^2}+y=0$. [5+5]

4.a) Show that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^n J_{n+1}(x)$.

b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n+1}$, $if\ m = n$. [5+5]

OR

5.a) Prove that $(n+1)\,P_{n+1}(x) = (2n+1)\,x\,P_n(x) - n\,P_{n-1}(x)$.

b) Prove that $J_n(x) = \frac{x}{2n}\left(J_{n-1}(x) + J_{n+1}(x)\right)$. [5+5]

6.a) Show that both the real part and the imaginary part of any analytic function satisfy Laplace's equation.

b) Evaluate $\int_{0}^{3+i} z^2\, dz$ along the path the real axis to 3 and then vertically to $3+i$.

[5+5]

OR

7.a) Find the analytic function $f(z) = u + i\,v$ where $u = \frac{\sin 2x}{(\cosh 2y - \cos 2x)}$.

b) Derive the Cauchy-Riemann equations if $f(z)$ is expressed in polar coordinates.

[5+5]

8. Use residue theorem to evaluate $\int_{0}^{2\pi} \frac{d\theta}{2+\cos\theta}$. [10]

OR

9. Use residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$. [10]

10.a) Determine and Plot the image of the region $-1 \le x \le 1$ and $-\pi \le y \le \pi$ under $w = e^z$.

b) Determine and Plot the image of the region $2 < |z| < 3$ and $|\arg z| < \frac{\pi}{4}$ under $w = z^2$. [5+5]

OR

11.a) Find and plot the rectangular region $0 \le x \le 2, 0 \le y \le 1$ under the transformation $w = \sqrt{2}\, e^{\frac{i\pi}{4}} z$.

b) Determine the bilinear transformation that maps the points $z_1 = -1$, $z_2 = i$, $z_3 = 1$ into the points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$ respectively. [5+5]

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