

Code No: 121AB

R15

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, May/June - 2017

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, AME, MIE, PTM, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**Part- A (25 Marks)**

- 1.a) Find the inverse of the matrix  $\text{diag}[a, b, c]$ ,  $a \neq 0, b \neq 0, c \neq 0$ . [2]
- b) Find the quadratic form corresponding to the symmetric matrix  $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -2 \\ 2 & -2 & 3 \end{bmatrix}$ . [3]
- c) Find C of the mean value theorem, if  $f(x) = x(x-1)(x-2)$  and  $a = 0, b = 0.5$ . [2]
- d) If  $u = \frac{x+y}{1-xy}$ ,  $v = \tan^{-1} x + \tan^{-1} y$  find  $\frac{\partial(u,v)}{\partial(x,y)}$ . [3]
- e) Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ . [2]
- f) Evaluate  $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx dy dz$ . [3]
- g) Find the general solution of  $(4D^2 + 4D + 1)y = 0$ . [2]
- h) Find  $\frac{1}{D^2 - 1} e^x$ . [3]
- i) Find  $L[te^{2t}]$ . [2]
- j) Find  $L^{-1}\left\{\frac{s}{(s-1)(s-2)}\right\}$ . [3]

**Part-B (50 Marks)**

2. Find for what values of  $\lambda$  the equations  $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$  have a solution and solve them in each case. [10]

**OR**

3. If  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$  verify Cayley-Hamilton theorem. Find  $A^{-1}$  using Cayley-Hamilton theorem. [10]

- 4.a) Verify Lagrange's mean value theorem for the function  $f(x) = e^x$  in  $[0, 1]$ .

- b) Expand  $\log \cos(x+h)$  in powers of  $h$  by Taylor's theorem. [5+5]

**OR**

- 5.a) If  $u$  and  $v$  are functions of  $x$  and  $y$  and  $J = \frac{\partial(u, v)}{\partial(x, y)}, J' = \frac{\partial(x, y)}{\partial(u, v)}$  then prove that  $JJ' = 1$ .

- b) Find the minimum values of  $x^2 + y^2 + z^2$  if  $x + y + z = 3a$ . [5+5]

6. Show that  $\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$ . [10]

**OR**

7. Change the order of integration and evaluate  $\int_0^a \int_{\frac{y}{a}}^{\sqrt{y/a}} (x^2 + y^2) dy dx$ . [10]

- 8.a) Solve  $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$ .

- b) If the surroundings are maintained at  $30^\circ\text{C}$  and the temperature of body cools from  $80^\circ\text{C}$  to  $60^\circ\text{C}$  in 12 minutes, find the temperature of body after 24 minutes. [5+5]

**OR**

- 9.a) Solve  $(D^2 + 3D + 2)y = e^{-x} + \cos x$ .

- b) Solve  $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$ . [5+5]

- 10.a) Find  $L^{-1} \left\{ \log \left( 1 + \frac{a^2}{s^2} \right) \right\}$

- b) Find  $L^{-1} \left\{ \frac{s}{(s^2 + 4)^2} \right\}$  by convolution theorem. [5+5]

**OR**

11. Solve  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1$  by Laplace transform. [10]