R15

Code No: 121AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year Examinations, August/September - 2016
MATHEMATICS-I
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$. [2]

b) If A is an n × n matrix and $A^2 = A$, then show that each Eigen value of A is 0 or 1. [3]

c) Give an example of a function that is continuous on [-1, 1] and for which mean value theorem does not hold with explanation. [2]

d) Find the maximum and minimum values of $x + y + z$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$. [3]

e) Evaluate $\int_0^\infty a^{-bx^2} dx$. [2]

f) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dy dx$ by changing into polar coordinates. [3]

g) Solve the differential equation $(hx + by + f)dy + (ax + hy + g)dx = 0$. [2]

h) Find the equation of the curve passing through the point (1,1) whose differential equation is $(y - yx)dx + (x + xy)dy = 0$. [3]

i) Find Laplace transform of $4 \sin(t - 3)\, u(t - 3)$. [2]

j) Express $f(t)$ in terms of Heavisides unit step function $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & t > 2 \end{cases}$ [3]

PART-B

(50 Marks)

2.a) Show that the two matrices A, $C^{-1}AC$ have the same latent roots.

b) For a matrix A = $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$. [5+5]

OR

3. Reduce the following quadratic form to canonical form and find its rank and signature $x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt$. [10]

4.a) Prove that $u = x + y + z, v = xy + yz + zx, w = x^2 + y^2 + z^2$ are functional dependent and find the relation between them.

b) If $x = u(1 - v); y = uv$ prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$. [5+5]

OR

5. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. [10]

6.a) Evaluate $\iint x^{m-1}y^{n-1}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

b) Evaluate $\int_0^\infty \frac{xdx}{(1+x^6)}$ using Γ- β functions. [5+5]

OR

7. Find the volume bounded by the cylinders $x^2+y^2=4$ and $z=0$. [10]

8. Solve $(D^2-4D+4)y=8x^2e^{2x}sin2x$. [10]

OR

9. A particle is executing simple harmonic motion of period T about a centre O and it passes through the position P(OP = b) with velocity v in the direction OP. Show that the time that elapses before it returns to P is $\frac{T}{\pi}tan^{-1}\frac{vT}{2\pi b}$. [10]

10. Solve the differential equation $\frac{d^2x}{dt^2}-4\frac{dx}{dt}-12x=e^{3t}$, given that $x(0)=1$ and $x'(0)=-2$ using Laplace transform. [10]

OR

11.a) Using Laplace transform, solve $(D^2+1)x=t\cos 2t$ given $x=0,\frac{dx}{dt}=0$ at $t=0$.

b) Using Convolution theorem, evaluate $L^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\}$. [5+5]

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