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Code No: 123AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, March - 2017

MATHEMATICS-III

(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the particular integral of $x^2\frac{d^2y}{dx^2}-6x\frac{dy}{dx}+10y=x^2$. [2]

b) Find the indicial equation of $x^2y''-2xy'-(x^2-2)y=0$. [3]

c) Prove that $\int_{-1}^{1}P_2^{\,2}(x)dx=\frac{2}{5}$. [2]

d) Prove that $J_1(0)=0$. [3]

e) Find the value of 'a' if cosax sinhy is harmonic. [2]

f) Find the analytic function whose real part is xy. [3]

g) Find the residue of $\frac{2z+3}{z^2-z-2}$ at z = -1. [2]

h) Expand $\frac{1}{3-z}$ when $|z|>3$ as Laurent series. [3]

i) Prove that $w=C+z$ where C is a complex constant is conformal at all points. [2]

j) Find the fixed points of $\frac{z+i}{1+iz}$. [3]

PART-B

(50 Marks)

2. Solve the differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$. [10]

OR

3. Solve the differential equation in series $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ around x = 0. [10]

4.a) Express x^2+x+1 in terms of Legendre Polynomials.

b) Prove that $\frac{d}{dx}\left(x^nJ_n(x)\right)=x^nJ_{n-1}(x)$. [5+5]

OR

5.a) Prove that $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+(n)P_{n-1}(x)$.

b) Prove that $J_4(x)=\left(\frac{48}{x^4}-\frac{8}{x^2}\right)J_1(x)-\left(\frac{24}{x^3}-1\right)J_0(x)$ [5+5]

6.a) Find the analytic function whose real part is e^{-x} (x sin y-y cos y).

b) Evaluate $\int_C \frac{dz}{(z-2)(z-4)}$ where C is $|z-3|=1/2$. [5+5]

OR

7.a) If f(z) is an analytic function then show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

b) Evaluate $\int_C \frac{dz}{(z^2-4)(z+1)}$ where C is $|z| = 3$. [5+5]

8. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using residue theorem. [10]

OR

9. Evaluate using residue theorem $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$. [10]

10.a) Under the transformation $w = \frac{z-i}{1-iz}$ find the image of the circle | z | = 1.

b) Find the image of $|z-3i|=3$ under the mapping w= $\frac{1}{z}$. [5+5]

OR

11.a) Find the image of the region bounded by the lines $x = 1$, $y = 1$, $x + y = 1$ under the transformation $w = z^2$.

b) Find the bilinear mapping which maps the points z = ∞, i, 0 into 0, i, ∞. [5+5]

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