Code No: 123AH **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, November/December - 2017

MATHEMATICS – III

(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Solve $\left(D^4+13D^2+36\right)y=0.$ [2]

b) Find the P.I of $\left(D^2+4\right)y=\cos 2x.$ [3]

c) Prove that $P_n^1(1)=\frac{1}{2}n(n+1).$ [2]

d) Prove that $\int xJ_0^2(x)dx=\frac{1}{2}x^2\left[J_0^2(x)-J_1^2(x)\right].$ [3]

e) If $u=e^x(x\cos y-y\sin y)$ then find analytic function of $f(z)$. [2]

f) Evaluate $\int_0^{1+i}(x^2-iy)\,dz$ along the path $y=x^2$. [3]

g) Expand $f(z)=\frac{z+3}{z(z^2-z-2)}$ for $|z|>2$. [2]

h) Find the residue of $f(z)=\frac{e^z}{(z-1)^2}$ at the singular point. [3]

i) Find the fixed points of $w=\frac{3z-2}{z+1}$. [2]

j) Prove that $w=\frac{1}{z}$ is circle preserving. [3]

PART-B

(50 Marks)

2.a) Solve $\left(D+2\right)\left(D-1\right)^2=e^{-2x}+2\sinh x.$

b) Solve $\left(1+x\right)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x).$ [5+5]

OR

3. Obtain the series solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$ [10]

4.a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \dfrac{2}{2n+1}, & if\ m = n \end{cases}$

b) Show that $\frac{2}{5}P_3(x) + \frac{3}{5}P_1(x) = x^3$. [5+5]

OR

5.a) Prove that $\frac{d}{dx}\left[J_n^{\ 2}(x) + J^2_{\ n+1}(x)\right] = 2\left[\frac{n}{x}J_n^{\ 2}(x) - \frac{(n+1)}{x}J^2_{\ n+1}(x)\right]$.

b) Show that $\left[J_{\frac{1}{2}}(x)\right]^2 + \left[J_{\frac{-1}{2}}(x)\right]^2 = \frac{2}{\pi x}$. [5+5]

6.a) Prove that the function of $f(z)$ defined by

$$f(z) = \frac{x^3(1+x) - y^3(1-i)}{x^2+y^2}, z \neq 0$$

$$= 0, \qquad z = 0$$

is continuous and C – R equations at the origin, yet $f'(0)$ does not exist.

b) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2 = 4\left|f'(z)\right|^2$. [5+5]

OR

7.a) Evaluate $\int_c \frac{z+4}{z^2+2z+5}dz$ where c is the circle.

i) $|z+1-i| = 2$

ii) $|z+1+i| = 2$

b) State and prove Cauchy's inequalities. [5+5]

8.a) State and prove residue theorem.

b) Evaluate $\int_0^{\pi} \frac{a d\theta}{a^2 + \sin^2\theta} (a > 0)$. [5+5]

OR

9.a) Evaluate $\int_0^{\infty} \frac{dx}{x^4 + a^4}$ $(a > 0)$.

b) Prove that $\int_0^{\infty} \frac{\sin mx}{x}dx = \frac{\pi}{2}$. [5+5]

10.a) Plot the image of $1 < |z| < 2$ under the transformation $w = 2iz + 1$.

b) Find the graph of the region $\frac{-\pi}{2} < x < \frac{\pi}{2}$, $1 < y < 2$ under the mapping $w = \sin z$. [5+5]

OR

11.a) Find the image of the region in the z-plane between the lines $y = 0$ and $y = \frac{\pi}{2}$ under the transformation $w = e^z$.

b) Find the bilinear transformation which maps the points $\infty, i, 0$ in the z-plane into $-1, -i, 1$ in the w-plane. [5+5]

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