Code No: 123BT **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, November/December - 2017
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Common to ECE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) A box contains nine cards numbered through 1 to 9, and B contains five cards numbered through 1 to 5. If a box is chosen at random, and a card is drawn which even numbered, what is the probability for the card to be from box A. [2]

b) Let a die be weighted such that the probability of getting numbers from 2 to 6 is that number of times of probability of getting a1. When the die thrown, what is the probability of getting an even or prime number occurs. [3]

c) Find the CDF of a random variable X, uniform over (-3, 3). [2]

d) The density of a random variable X is given as $f(x)= K[U(x)-U(x-4)]+0.25\delta(x-2)$. Find the probability of $X \leq 3$. [3]

e) X and Y are discrete random variables and their joint occurrence is given as

X\Y	1	2	3
1	1/18	1/9	1/6
2	1/9	1/18	1/9
3	1/6	1/6	1/18

Find the Conditional Mean of X, given Y=2. [2]

f) X and Y are two uncorrelated random variables with same variance. If the random variables U=X+ kY and $V=X+(\sigma x/\sigma y)Y$ are uncorrelated, find K. [3]

g) State and prove the Periodicity Property of Auto Correlation function of a Stationary Random Process. [2]

h) If X(t) is a Gaussian Random Process with a mean 2 and exp $(-0.2|\tau|)$. Find the Probability of $X(1) \leq 1$. [3]

i) Verify that the cross spectral density of two uncorrelated stationary random processes is an impulse function. [2]

j) The output of a filter is given by Y(t)=X(t+T)+X(t-T), where X(t) is a WSS process, power spectral density $S_{xx}(w)$, and T is a constant. Find the power spectrum of Y(t). [3]

PART-B

(50 Marks)

2.a) Consider the experiment of tossing two dice simultaneously. If X denotes the sum of two faces, find the probability for $X \leq 6$.

b) A fair coin is tossed 4 times. Find the probability for the longest string of heads appearing to be three as a result of the above experiment.

c) In certain college, 25% of the boys and 10% of the girls are studying Mathematics. The girls constitute 60% of the student body. If a student is selected at random and studying mathematics, determine the probability that the student is a girl. [3+3+4]

OR

3.a) Coin A has a probability of head =1/4 and coin B is a fair coin. Each coin is flipped four times. If X is the number of heads resulting from coin and Y denotes the same from coin B, what is the probability for X=Y?

b) A dice is thrown 6 times. Find the probability that a face 3 will occur at least two times. [6+4]

4.a) Find the Moment generating function of a uniform random variable distribute over (A, B) and find its first and second moments about origin, from the Moment generating function.

b) A random variable X has a mean of 10 and variance of 9. Find the lower bound on the probability of (5<X<15). [5+5]

OR

5.a) Find the Moment generating function of a random variable X with density function

$$f(x) = \begin{Bmatrix} x, & for\ 0 \le x \le 1 \\ 2-x, & for\ 1 \le x \le 2 \\ 0, & else\ where \end{Bmatrix}$$

b) If X is a Gaussian random variable N(m, σ^2), find the density of Y=PX+Q, where P and Q are constants. [5+5]

6.a) If X_1, X_2, X_3, - - - - - X_n are 'n' number of independent and Identically distributed random variables, such that X_k = 1 with a probability 1/2; = -1 with a probability 1/2. Find the Characteristic Function of the random Variable Y= $X_1+X_2+X_3+$ - - - + X_n.

b) If Independent Random Variables X and Y both of zero mean, have variance 20 and 8 respectively, find the correlation coefficient between the random Variables X+Y and X-Y. [5+5]

OR

7.a) Let X=Cosθ and Y=Sinθ , be two random variables, where θ is also a uniform random variable over (0,2π). Show that X and Y are uncorrelated and not independent.

b) If X is a random variable with mean 3 and variance 2, verify that the random Variables 'X' and Y= -6X+22 are orthogonal. [6+4]

8.a) X(t) is a random process with mean =3 and Autocorrelation function $R_{xx}(\tau)$ =10.[exp(- 0.3|τ|)+2]. Find the second central Moment of the random variable Y=X(3)-X(5).

b) X(t)=2ACos(Wct+2θ) is a random Process, where 'θ' is a uniform random variable, over (0,2π). Check the process for mean ergodicity. [5+5]

OR

9.a) A Random Process X(t)=A.Cos (2π fc t) , where A is a Gaussian Random Variable with zero mean and unity variance, is applied to an ideal integrator, that integrates with respect to 't', over (0,t). Check the output of the integrator for stationarity.

b) A random Process is defined as X(t)=3.Cos(2πt+Y), where Y is a random Variable with p(Y=0)=p(Y=π)=1/2.Find the mean and Variance of the Random Variable X(2). [5+5]

10.a) Find and plot the Autocorrelation function of (i) Wide band white noise (ii) Band Pass White noise.

b) Derive the expression for the Cross Spectral Density of the input Process X(t) and the output process Y(t) of an LTI system in terms of its Transfer function.

[5+5]

OR

11.a) Compare and contrast Auto and cross correlations.

b) If $Y(t) = A.Cos(w_0 t+\theta)+N(t)$, where '$\theta$' is a uniform random variable over $(-\pi,\pi)$, and N(t) is a band limited Gaussian white noise process with PSD=K/2. If 'θ' and N(t) are independent, find the PSD of Y(t). [4+6]

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