# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD 

## B.Tech I Year I Semester Examinations, May - 2018 <br> MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

## Time: 3 hours

Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(25 Marks)
1.a) Find an integrating factor for the following equation $\frac{d y}{d x}=e^{2 x}+y-1$.
b) Find the solution of $\frac{d y}{d x}=-\frac{x}{y}$ at $\mathrm{x}=1$ and $\mathrm{y}=\sqrt{3}$.
c) Find the value of $\alpha$ such that the vectors (1, 1, 0), (1, $\alpha, 0)$ and (1, 1, 1) are linearly dependent.

$$
\begin{equation*}
2 x-3 y+5 z=1 \tag{2}
\end{equation*}
$$

d) Determine whether the system of equations is consistent $3 x+y-z=2$

$$
\begin{equation*}
x+4 y-6 z=1 \tag{3}
\end{equation*}
$$

e) If $\lambda$ is the Eigen value of a matrix $A$ then derive the Eigen value of (adjoint $A$ ).
f) Taking A as a $2 \times 2$ matrix show that the Eigen values of $\mathrm{A}=$ the trace of A .
g) If $u=x^{y}$ show that $\frac{\partial^{3} u}{\partial x^{2} \partial y}=\frac{\partial^{3} u}{\partial x \partial y \partial x}$.
h) Find the stationary values of $x y(a-x-y)$.
i) Eliminate the arbitrary function $f$ from the equation and form the partial differential equation $z=x y+f\left(x^{2}+y^{2}\right)$.
j) Eliminate the constants $a$ and $b$ from the equation: $z=(y+a)(x+b)$.

## PART-B

(50 Marks)
2.a) Solve the Following differential equations:

$$
\begin{equation*}
y^{\prime \prime}-2 y^{\prime}+y=t e^{t}+4, \quad y(0)=1, \quad y^{\prime}(0)=1 \tag{5+5}
\end{equation*}
$$

b) Find the orthogonal trajectories for the family of curves $r^{n} \sin n \theta=a^{n}$.

OR
3.a) In an $\mathrm{L}-\mathrm{R}$ circuit an e.m.f. of 10 sin t volts is applied. If $\mathrm{I}(0)=0$, find the current $\mathrm{I}(\mathrm{t})$ in the circuit at any time $t$.
b) Solve the Following differential equation $y^{\prime \prime}+2 y^{\prime}+5 y=4 e^{-t} \cos 2 t, y(0)=1, y^{\prime}(0)=0$.
4.a) Find an LU factorization for the matrix $\left[\begin{array}{cc}1 & 2 \\ -3 & -1\end{array}\right]$
b) In the following equations determine, for what value of " $k$ " if any will the systems have i) unique solution ii) no solution iii) Infinitely many solutions

$$
\begin{align*}
& k x+2 y=3 \\
& 2 x-4 y=-6 \tag{5+5}
\end{align*}
$$

## OR

5.a) Use either the Gaussian Elimination or the Gauss Jordan method to solve

$$
\begin{gather*}
x+2 y-3 z=9 \\
2 x-y+z=0 \\
4 x-y+z=4 \tag{5+5}
\end{gather*}
$$

b) Using the theory of matrices, find the point such that the line of intersection of the planes $3 x+2 y+z=-1$ and $2 x-y+4 z=5$ cuts the plane $x+y+z=4$.
6.a) Obtain the Eigen values of the following matrix $A=\left[\begin{array}{lll}2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2\end{array}\right]$ and verify whether its Eigen vectors are orthogonal.
b) Show that 0 is an Eigen value of a matrix $A$ if and only if it is singular.

## OR

7.a) If $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0\end{array}\right]$ then show that $A^{n}=A^{n-2}+A^{2}-I$ for $n \geq 3$. Hence find $A^{50}$.
b) Show that the matrix $A=\left[\begin{array}{ccc}2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1\end{array}\right]$ is not similar to a diagonal matrix.
8.a) If $\sin u=\frac{x^{2} y^{2}}{x+y}$ show that $x \frac{\partial u}{\partial x}+y \frac{\partial u}{\partial y}=3 \tan u$.
b) If $f(0)=0$ and $f^{\prime}(x)=\frac{1}{1+x^{2}}$ then using Jacobians show that $f(x)+f(y)=f\left(\frac{x+y}{1-x y}\right)$.
[5+5]

## OR

9.a) Expand $e^{x} \cos y$ in powers of $x$ and $\left(y-\frac{\pi}{2}\right) 0$.
b) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.
10. Find the general integrals of the linear partial differential equations
a) $y^{2} p-x y q=x(z-2 y)$
b) $(y+z x) p-(x+y z) q=x^{2}-y^{2}$.

OR
11. Find complete integrals of the following equations
a) $p+q=p q$
b) $p^{2} q\left(x^{2}+y^{2}\right)=p^{2}+q$.

