

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, December – 2019/January - 2020

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

- 1.a) Find $L[e^{2t} \cdot t]$. [2]
- b) Find inverse Laplace of $\frac{1}{s^2 + 2s + 2}$. [3]
- c) Evaluate $\int_0^{\pi/2} \sin^{3/4} x \cos^{3/4} x dx$ using Beta -Gamma functions. [2]
- d) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, using the relation $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [3]
- e) Evaluate $\int_0^1 \int_0^1 \int_0^1 dx dy dz$ [2]
- f) Evaluate $\int_0^2 \int_0^{\sqrt{2x}} dy dx$. [3]
- g) Find the directional derivative of $x^2 + y^2 + z^2 = r^2$, at the point (0, r, 0) in the direction of \bar{j} [2]
- h) Find Curl (grad f) where $f = x^2 + y^2 - z$. [3]
- i) State Green's theorem in a plane. [2]
- j) Find the work done by $\bar{F} = 2x\bar{i} + 2y\bar{j} + 3z\bar{k}$ in moving a particle from (-1, 2, 1) to (2, 3, 4) along the line joining them. [3]

PART-B

(50 Marks)

- 2.a) Find inverse Laplace transform $L^{-1}\left\{\log\left(\frac{s^2+1}{(s-1)^2}\right)\right\}$
- b) Find $L\{te^{2t} \sin 3t\}$ [5+5]
- OR**
3. Using Laplace transform solve $(D^2 + 3D + 2)y = 3$, $y(0) = y'(0) = 1$. [10]
- 4.a) Prove the relation between Beta and Gamma functions.
- b) If $\frac{\beta(m, n)}{k} = \frac{\beta(m-1, n+1)}{p}$ find k+p in terms of m and n. [5+5]

OR

5.a) Prove that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

b) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}} dx = \frac{\beta(m,n)}{a^n(1+a)^m}$. [5+5]

6.a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates.

b) Evaluate $\iiint_V (xy + yz + zx) dx dy dz$ where V is the region of space bounded by $x=0, x=1, y=0, y=2, z=0, z=3$. [5+5]

OR

7.a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy dy dx$.

b) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dx dy dz$. [5+5]

8.a) Prove that $\nabla(\bar{A} \cdot \bar{B}) = (\bar{B} \cdot \nabla)\bar{A} + (\bar{A} \cdot \nabla)\bar{B} + \bar{B} \times (\nabla \times \bar{A}) + \bar{A} \times (\nabla \times \bar{B})$.

b) Evaluate the angle between the normals of the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3). [5+5]

OR

9.a) Find the unit normal vector to the surface $x^2y + 2xz = 4$ at the point (2,-2,3). Also find the directional derivative of the surface in the direction normal to the surface $x \log z - y^2 = 1$ at (-1, 2, 1).

b) For any vector field \bar{V} , prove that $\text{Div Curl } \bar{V} = 0$. [5+5]

10. Verify Divergence Theorem for $\bar{F} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ over the surface S of the solid cut off by the plane $x+y+z=a$ in the 1st octant. [10]

OR

11.a) Find the work done by the force $\bar{F} = (xy)\bar{i} - z\bar{j} + (x^2)\bar{k}$ along the curve $x = t^2; y = 2t; z = t^3$ from $t=0$ to $t=1$.

b) Apply Stokes Theorem to evaluate $\int_C \bar{F} \cdot d\bar{R}$, where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$ and $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$. [5+5]

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