R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, May/June - 2017

MATHEMATICS-II

(Common to CE, ME, MCT, MMT, MIE, CEE, MSNT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

Part- A (25 Marks)

1.a) Find $\lim_{t\to 0} f(t)$, if $L(f(t)) = \frac{s}{s^2+w^2}$. [2]

b) Find the inverse Laplace Transform of $\frac{s+5}{(s+1)(s+3)}$. [3]

c) Find the value of $\int_0^\infty \frac{dx}{1+x^4}$. [2]

d) Evaluate $\int_0^1 x^{11}(1-x)^{16}\,dx$. [3]

e) Find the area enclosed between the parabola $y = x^2$ and the line $y = x$. [2]

f) Evaluate $\int_0^{\frac{\pi}{4}} \cos^2 x\,dx$. [3]

g) Find the magnitude of the gradient of the function $f(x,y,z) = xyz^3$ at (1, 0, 2). [2]

h) The velocity vector in 2-dimensional field is $\overline{V} = 2xy\bar{i} + (2y^2 - x^2)\bar{j}$. Find the $curl\,\overline{V}$. [3]

i) Find the Curl of the gradient of the scalar field $V = 2x^2y + 3y^2z + 4z^2x$. [2]

j) Find the divergence of the vector field $\overline{A}$ at (1, -1, 1) $\overline{A} = x^2z\bar{i} + xy\bar{j} - yz^2\bar{k}$. [3]

Part-B (50 Marks)

2.a) State and prove the second shifting theorem of Laplace Transform.

b) Find $L(F(t))$ if $F(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$ [5+5]

OR

3.a) Find the Laplace Transform of $F(t) = a + bt + \frac{c}{\sqrt{t}}$.

b) Solve using Laplace Transform $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3\cos 3t - 11\sin 3t,\ \ y(0) = 0,\ \ y'(0) = 16.$

[5+5]

4.a) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\ d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\ d\theta = \frac{\pi}{\sqrt{2}}$.

b) Evaluate $\frac{\beta(m+1,n)}{\beta(m,n)}$. [5+5]

OR

5. Show that $\Gamma m\, \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m),\ m > 0.$ [10]

6. Find the mass, center of gravity and moment of inertia relative to the x-axis, y-axis and origin of a rectangle $0 \le x \le 4,\ \ 0 \le y \le 2$ having the mass density function $f(x, y) = xy.$

[10]

OR

7. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\ dy\ dx$ and integrate it. [10]

8.a) Show that $div\left(r^n \bar{r}\right) = (n+3)r^{-n}$.

b) If $\bar{u} = \frac{1}{r}\bar{r}$, find $grad\left(div\,\bar{u}\right)$. [5+5]

OR

9.a) Show that $div\left(\bar{A} \times \bar{B}\right) = \bar{B}.curl\bar{A} - \bar{A}.curl\bar{B}.$

b) Find the gradient of the Scalar function $f(x, y, z) = x^2y^2 + xy^2 - z^2$ at (3, 1, 1). [5+5]

10. Verify the Gauss's divergence theorem for $\bar{F} = \left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$ over the rectangular parallelepiped $0 \le x \le a,\ \ 0 \le y \le b,\ \ 0 \le z \le c.$ [10]

OR

11. Evaluate $\oint_c x^2dx + 2ydy - dz$ by Stoke's theorem where C is the curve $x^2 + y^2 = 4, z = 2.$

[10]

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