Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, April/May - 2018
MATHEMATICS – IV
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Show that $u = \frac{x}{x^2+y^2}$ is harmonic [2]

b) Write Cauchy-Riemann equations in polar form. [3]

c) Expand $f(z) = sinz$ in Taylor's series about $z = \frac{\pi}{4}$ [2]

d) Find residue of $f(z) = \frac{z}{z^2+1}$ at its poles [3]

e) Find image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$ [2]

f) Determine the region of w-plane into which the region is mapped by the transformation $w = z^2|z-1| = 2$. [3]

g) Find the value b_n of the Fourier series of the function$f(x) = x^2 - 2$, when $-2 \le x \le 2$ [2]

h) Find the Fourier sine transformation of $2e^{-5x} + 5e^{-2x}$ [3]

i) Classify the equation $3\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - 4 = 0$ [2]

j) Write the one dimensional Heat equation in steady state. [3]

PART-B

(50 Marks)

2.a) Discuss the continuity of $f(x,y) = \begin{cases} \frac{2xy(x+y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

b) Construct the analytic function $f(z)$, whose real part is $e^x \cos y$. [5+5]

OR

3.a) If $f(z) = u + iv$ is an analytic function of z and if $u - v = e^x(cosy - siny)$ find f(z) in terms of z

b) if $u(x, y)$ and v(x, y) are harmonic functions in a region R, Prove that the function $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is an analytic function. [5+5]

4.a) Evaluate $\int_c (x-2y)dx + (y^2 - x^2)dy$ where c is the boundary of the first quadrant of the circle $x^2 + y^2 = 4$

b) Evaluate $\int_c \frac{1}{z^8(z+4)} dz$, where c is the circle $|z| = 2$. [5+5]

OR

5.a) Obtain the expansion for $sin\left[\frac{1}{z-1}\right]$ which is valid in $1 < |z| < \infty$

b) Evaluate $\int_c \frac{(2z+1)^2}{z^8(4z^3+z)} dz$ over a unit circle C. [5+5]

6. Prove that $\int_0^{2\pi} \frac{sin^2\theta}{a+b\ cos\theta} d\theta = \frac{2\pi}{b^2}\left[a - \sqrt{a^2 - b^2}\right], a > b > 0$ [10]

OR

7. Find the bilinear transformation that maps the points 1, i, -1 into the points 2, i, -2 respectively. [10]

8.a) Obtain the Fourier series for the function $f(x) = |\sin x|$ in (-π, π)

b) Find Fourier Sine transformation of $e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin(\alpha x)}{1+x^2} dx$ [5+5]

OR

9.a) Find the Half range cosine series for $f(x) = x(2-x)$ in $0 \le x \le 2$

b) Find the inverse Fourier sine transform of $F_s(p) = \frac{e^{-ap}}{p}$ [5+5]

10. Show that the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod Without radiation, subject to the following conditions:

a) u is not infinite for $t \to \infty$

b) $\frac{\partial u}{\partial x} = 0\ for\ x = 0\ and\ x = l$

c) $u = lx - x^2\ for\ t = 0\ and\ x = l.$ [10]

OR

11.a) Solve by the method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

b) Solve by the method of separation of variables $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = 3e^{-y} - e^{-5y}$ when $x = 0$. [5+5]

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