Code No: 133BD



(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT) Time: 3 Hours Max. Marks: 75

Note: This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

R16

- 1.a) State the necessary and sufficient conditions for a function f(z) to be analytic. [2]
 - b) Show that the function f(z) = xy + iy is everywhere continuous but is not analytic. [3]

c) Show that
$$f(z) = \frac{1}{1-e^z}$$
 has a simple pole at $z = 2\pi i$. [2]

- d) State Cauchy's integral formula and use it to evaluate $\oint_C \frac{z^2+4}{z-3} dz$ where *C* is the circle |z| = 5. [3]
- e) Find the fixed points of the mapping w = z + 2i. [2]
- f) Find the residues at the poles of the function $f(z) = \frac{2z+1}{(z-1)^2}$, $C: |z| \le 4$. [3]

g) If $f(x) = x^3$ in $[-\pi, \pi]$, find the Fourier coefficient b_n . [2]

- h) Find f(x) if its finite sine transform is given by $\overline{f_s}(s) = \frac{1+\cos s\pi}{s\pi}$ where $0 < x < \pi$, s = 1, 2, 3, ... [3]
- i) Classify the PDE: $xu_{xx} u_{xy} + yu_{yy} = 1$.
- j) Write the possible three solutions of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$. [3]

PART-B

(50 Marks)

[2]

- 2.a) Define analyticity of a function. Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin although the C-R equations are satisfied at that point.
 - b) Find the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$, when $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$. [5+5] **OR**
- 3.a) Show that both the real and imaginary parts of an analytic function are harmonic.
- b) If f(z) = u + iv be an analytic function of z and if $u v = (x y)(x^2 + 4xy + y^2)$ find f(z) in terms of z. [5+5]

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- 4.a) State Cauchy integral theorem and use it to evaluate the integral $\int_C \frac{e^{2z}}{(z-1)^2(z-3)} dz$ where *C* is the circle |z| = 4.
 - b) If $\Phi(a) = \int_c \frac{3z^2 + 7z + 1}{z a} dz$, where *C* is the circle $x^2 + y^2 = 4$, find $\Phi(3)$, $\Phi'(1 i)$ and $\Phi''(1 i)$. [5+5]

OR

- 5.a) Expand $f(z) = \frac{1}{z^2 4z + 3}$ in the region 1 < |z| < 3. Also name the series so obtained.
 - b) Find the nature and location of the singularities of the function $f(z) = \frac{e^{2z}}{(z-2)^4}$ by finding its Laurent's series expansion. [5+5]

6. State Residues theorem. Evaluate the integral by contour integration: $\int_0^{\pi} \frac{d\theta}{13+5\cos\theta}$. [10] **OR**

- 7.a) Find the residue of $f(z) = \frac{z^3}{z^2 1}$ at $z = \infty$.
 - b) Define bilinear transformation. Find the bilinear transformation which maps the points z = 1, i, -1 onto the points w = i, 0, -i and hence find the image of |z| < 1. [5+5]

8.a) Find the Fourier series for the function
$$f(x) = \frac{\pi x}{2}$$
 in $0 \le x \le 2$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a \end{cases}$. Hence prove that $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$. [5+5]

OR

- 9.a) Develop $f(x) = \begin{cases} 2, & -2 < x < 0, \\ x, & 0 < x < 2 \end{cases}$ in a series of sines and cosines and deduce the series for π^2 .
- b) Find the Fourier cosine transform of $f(x) = e^{-x}$, x > 0. [5+5]
- 10. The ends A and B of a rod 20 cm long have the temperature at 30^{0} C and 80^{0} C until steady state conditions prevail. The temperature at the ends are suddenly changed to 40^{0} C and 60^{0} C respectively. Find the temperature distribution in the rod at time *t*. [10]

OR

11. Write down one dimensional wave equation. A string is stretched and fastened to two points l cm apart. Motion is started by displacing the string in a sinusoidal arch of height y_0 and then released from rest at time t = 0. Find the displacement at point x and at any time t. [10]

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