

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the Laplace transform of unit step function.
(b) Find the $L[1-\cos t]$. [7+8]
2. (a) Find $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$.
(b) Solve $(D^2 + 2D - 3)y = \sin t$; $y(0) = y'(0) = 0$, using Laplace transforms. [7+8]
3. (a) Expand $\frac{\pi^2}{12} - \frac{x^2}{4}$ as a Fourier series in $(-\pi, \pi)$.
(b) Find the Half range cosine series of $f(x) = 4x$ in $[0, 2]$. [8+7]
4. (a) Prove that $F\{x^n f(x)\} = (-i)^n \frac{d^n}{dp^n}[F(p)]$
(b) Find finite Fourier cosine transform of $f(x) = x + a$ for $0 < x < \pi$. [8+7]
5. (a) Form the P.D.E. by eliminating ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$.
(b) Solve $(y + z)p - (z + x)q = x - y$. [7+8]
6. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n\pi x}{l}$. [15]
7. (a) Find u_2, u_3 if $\bar{u}(z) = \frac{(2z^2 + 5z + 14)}{(z-1)^4}$.
(b) Find the inverse Z- transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. [7+8]
8. (a) Show that $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx$
(b) Evaluate $4 \int_0^{\infty} \frac{x^2}{1+x^4} dx$ using beta gamma function. [8+7]

