

**MATHEMATICS – II**

(Common to All Branches)

**Time: 3 hours****Max. Marks: 75**

**Answer any FIVE Questions**  
**All Questions carry equal marks**

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1. (a) Find the Laplace transform of  $e^t \cos 4t \sin t$   
 (b) Find the Laplace transform of Dirac-delta function [7+8]
  
2. (a) Find  $L^{-1} \left[ \frac{s^2 + 2s - 4}{(s^2 + 9)(s - 5)} \right]$   
 (b) Using Laplace transform method solve  $(D^2 + 5D - 6)y = t^2 e^{-t}$ ,  $y(0) = a$ ,  $y'(0) = b$ . [7+8]
  
3. (a) Find the Half range sine series of  $f(x) = x(l-x)$ , in  $0 < x < l$ . Hence Evaluate  $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \dots$   
 (b) Expand  $\cosh x$  as a Fourier series in  $(-\pi, \pi)$  [8+7]
  
4. (a) Using Fourier integral show that  $e^{-bx} = \frac{2b}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + b^2} d\lambda$ , ( $b > 0$ )  
 (b) Find the inverse Fourier transform of  $F(s) = e^{-|s|y}$  [8+7]
  
5. (a) Solve the PDE  $(x^2 + y^2)(p^2 + q^2) = 1$   
 (b) Solve the PDE  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$  [7+8]
  
6. A tightly Stretched string with fixed end points  $x = 0$  and  $x = l$  is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity  $\lambda x(l-x)$ , find the displacement of the string at any distance  $x$  from one end at any time [15]
  
7. (a) Find  $Z^{-1} \left[ \frac{8z - z^3}{(4-z)^3} \right]$   
 (b) Find  $Z \left[ \cos \left( \frac{n\pi}{2} + \theta \right) \right]$  [8+7]
  
8. (a) Prove that  $\int_a^b (x-b)^{m-1} (a-x)^{n-1} dx = (a-b)^{m+n-1} \beta(m, n)$   
 (b) Evaluate  $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx$  [8+7]

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