

Answer any FIVE Questions  
All Questions carry equal marks

\* \* \* \* \*

1. (a) Find  $L \left\{ \int_0^t t e^{-t} \sin 2tdt \right\}$ .  
(b) Find the Laplace transform of Dirac-delta function. [7+8]
2. (a) Using the Convolution theorem, then find  $L^{-1} \left\{ \frac{1}{S(S+1)(S+2)} \right\}$ .  
(b) Solve  $(D^2 + 2D - 3)y = \cos t; y(0) = y'(0) = 0$ , using Laplace transforms. [7+8]
3. (a) Obtain the Fourier series for  $f(x) = \begin{cases} 2, & -2 < x \leq 0 \\ x, & 0 \leq x < 2 \end{cases}$   
(b) Find the Half range cosine series of  $f(x) = x/2$  in  $[0, \pi]$ . [8+7]
4. (a) Find the Fourier sine transform of  $f(x) = x^{n-1}$ .  
(b) Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$   
Hence Deduce that  $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$  and  $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$  [7+8]
5. (a) Form the P.D.E. by eliminating a, b, c from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$   
(b) Solve  $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$  [7+8]
6. Find the physically feasible solution of one-dimensional heat flow equation:  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ . [15]
7. (a) Solve  $u_{n+2} - u_n = 2^n$  if  $u_0 = 0$  and  $u_1 = 1$   
(b) Find  $Z^{-1} \left[ \frac{3z^2 + z}{(5z-1)(5z+2)} \right]$  [8+7]
8. (a) Show that  $\beta(m, n) = \int_0^{\pi/2} \sin 2^{m-1} \theta \cos 2^{n-1} \theta d\theta$   
(b) Evaluate  $\int_0^{\pi/2} \sin^5 \theta d\theta$  using beta gamma function [8+7]

\*\*\*\*\*