**Subject Code: R13202/R13** **Set No - 1**

# I B. Tech II Semester Supplementary Examinations Feb. - 2015

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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## PART-A

1.(i) Define Echelon form of a matrix?

(ii) Prove that zero is an eigen value of a matrix iff it is singular?

(iii) Find the complete area of the curve $a^2y^2 = x^3(2a - x)$

(iv) Evaluate $\int_0^1 \sqrt[3]{\log\frac{1}{x}}\,dx$

(v) Write the physical significance of grad φ.

(vi) Write the physical interpretation of surface integrals.

[4+4+4+4+3+3]

## PART- B

2.(a) Evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ by using Green's theorem where C is the boundary of the surface in the xy plane enclosed by x-axis and the semi-circle.

(b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form.

[8+8]

3.(a) Solve by Gauss – Seidal method, the equations

$9x - 2y + z - t = 50$

$x - 7y + 3z + t = 20$

$-2x + 2y + 7z + 2t = 22$

$x + y - 2z + 6t = 18$

(b) Evaluate the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and (3, 3, -3).

[8+8]

4.(a) Find the surface got by rotating one loop of the curve $r^2 = a^2\cos 2\theta$.

(b) Determine the rank of a matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to normal form.

[8+8]

5.(a) Trace the curve $a^2y^2 - x^2(a^2 - x^2) = 0$.

(b) Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$. [8+8]

6.(a) If $f, g$ are scalar fields, show that $\nabla f \times \nabla g$ is solenoidal.

(b) Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis.

[6+10]

7.(a) Determine the natural frequencies and normal modes of vibrating system for which $M = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $K = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$.

(b) Test for consistency and solve $x + y = 0$; $y + z = 0$; $z + x = 0$.

[8+8]

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**Subject Code: R13202/R13** **Set No - 2**

## I B. Tech II Semester Supplementary Examinations Feb. - 2015

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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**PART-A**

1.(i) Define normal form of a matrix?

(ii) Prove that $\lambda$ is an eigen value of a non-singular matrix $A$, show that $\frac{|A|}{\lambda}$ is an eigen value of $Adj.\ A$?

(iii) Find the perimeter of the curve $y^2 + x^2 = a$?

(iv) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta$

(v) Define Directional Derivative.

(vi) Write the statement of Stoke's theorem.

[4+4+4+4+3+3]

**PART- B**

2.(a) Reduce $A$ to canonical form and find its rank, if $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

(b) Trace the curve $y = \frac{x^2+1}{x^2-1}$

[8+8]

3.(a) Evaluate $\iiint_V (2x + y)dv$ where $\boldsymbol{V}$ is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 2\ and\ z = 0.$

(b) Show that $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{\beta(m,n)}{a^n b^m}$

[8+8]

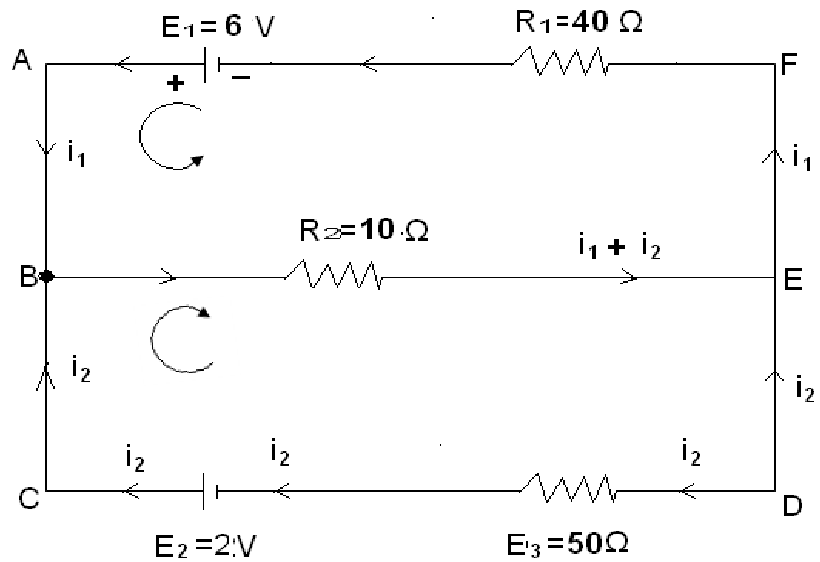
4. Find a non-singular matrix $\boldsymbol{P}$ such that $\boldsymbol{A}$ is diagonalizable, where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

Hence diagonalise $\boldsymbol{A}$.

[16]

5.(a) If $A$ and $B$ are $n$ rowed square matrices and if $A$ is invertible show that $A^{-1}B$ and $BA^{-1}$ have same Eigen Values.

(b) Find the current in each cell considering the circuit given below




[8+8]

6.(a) Find the moment of inertial of the area of a circle A of radius R relative to the centre O.

(b) If $A = z^2 i + x^2 j - y^2 zk$, and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0 \; and \; z = -5$, evaluate $\iint_S \boldsymbol{A.n} \, ds$

[6+10]

7.(a) Find the total work done by a force $F = 2xyi - 4zj + 5xk$ along the curve $x = t^2, y = 2t + 1, \; z = t^3$, from the points $t = 1, t = 2$.

(b) Evaluate $\int_0^2 (8 - x^3)^{-\frac{1}{3}} dx$ using $\beta - \Gamma$ functions.

[8+8]

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**Subject Code: R13202/R13** **Set No - 3**

**I B. Tech II Semester Supplementary Examinations Feb. - 2015**

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
*****

## PART-A

1.(i) Define Rank of a matrix?

(ii) Prove that every Hermitian matrix can be written as $A+iB$, where $A$ is real and symmetric and $B$ is real and skew-symmetric.

(iii) Define Gamma function.

(iv) Evaluate $\nabla^2(\log r)$

(v) Define asymptote.

(vi) Write the statement of Divergence theorem.

[4+4+4+4+3+3]

## PART- B

2.(a) Reduce $A$ to normal form and find its rank, if $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

(b) Trace the curve $r = a\sin 2\theta$

[8+8]

3.(a) Evaluate $\iiint_V f\,dv$ where $f = 45x^2y$ and $\boldsymbol{V}$ is the closed region bounded by the planes $4x + 2y + z = 8,\ x = 0,\ y = 0\ and\ z = 0.$

(b) Solve the system of equations
$10x - y - z = 13;\ \ x + 10y + z = 36;\ -x - y + 10z = 35.$

[8+8]

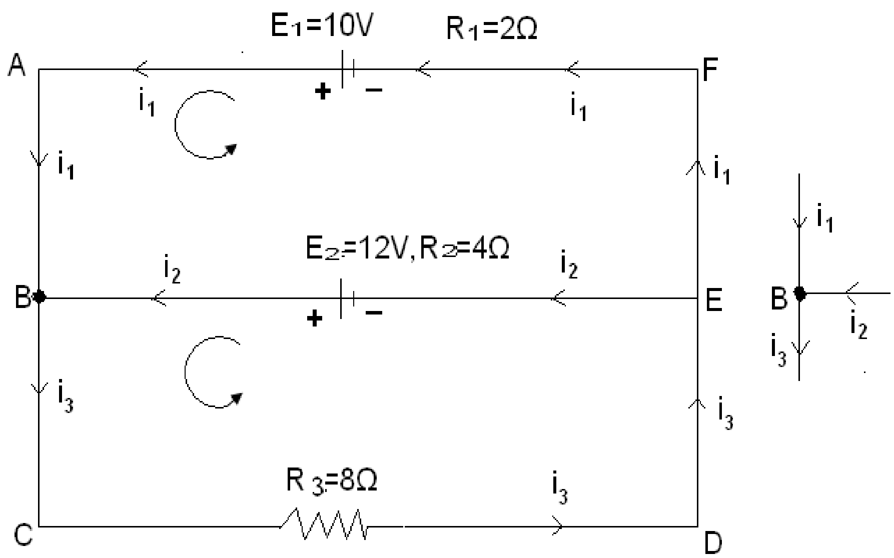
4. Find a non-singular matrix $\boldsymbol{P}$ such that $\boldsymbol{A}$ is diagonalizable, where $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

Hence diagonalzie A.

[16]

5.(a) If $A$ and $B$ are $n$ rowed square matrices and if $A$ is invertible show that $A^{-1}B$ and $BA^{-1}$ have same Eigen Values.

(b) Find the current in each cell considering the circuit given below




[8+8]

6.(a) Find the perimeter of the loop of the curve $3ay^2 = x^2(a - x)$.

(b) Find the work done in moving a particle in the force field $F = 2x^2 i + (2yz - x)j - yk$, along

(i) the straight line from (0, 0, 0) to (3, 1, 2)

(ii) the space curve $x = 3t^2, y = t, z = 3t^2 - t$ from $t=0$ to $t=1$.

[6+10]

7.(a) Test the following system for consistency and if consistent solve it

$u + 2v + 2w = 1; 2u + v + w = 2; 3u + 2v + 2w = 3; v + w = 0$

(b) Evaluate $\int_0^{\frac{\pi}{8}} \sin^4 8\theta \cos^6 4\theta \, d\theta$.

[8+8]

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**Subject Code: R13202/R13** **Set No - 4**

**I B. Tech II Semester Supplementary Examinations Feb. - 2015**

# MATHEMATICS-III

**(Common to All Branches)**

**Time: 3 hours** **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
*****

## PART-A

1.(i) Write the *Krichoff's* laws.
(ii) Write the statement of *Cayley-Hamilton* theorem.
(iii) Write the relation between *Beta* and *Gamma* functions.
(iv) State *irrotational* and *solenoidal* vectors
(v) Write the statement of *Sylvester's* law of inertial.
(vi) Define latent roots and vectors.

[4+4+4+4+3+3]

## PART- B

2.(a) Evaluate $\oint_c (2xy - x^2)dx + (x + y^2)dy$ where C is the closed curve in xy plane bounded by the curves $y = x^2$ $and$ $y^2 = x$..

(b) Determine the Eigen values and Eigen vectors of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

[8+8]

3.(a) Solve the equations
$2x - y + 3z - 9 = 0$
$x + y + z = 6$
$x - y + z - 2 = 0$

(b) Find the area of the curve $r^2 = a^2 \sin 2\theta$.

[8+8]

4.(a) Find the unit normal to the surface $xy + yz + zx = 3$ at the point *(1, 1, 1).*

(b) Determine the rank of a matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to normal form.

[8+8]

5.(a) Trace the curve $y(a^2 + x^2) = a^2 x$.

(b) Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} dx$.

[8+8]

6.(a) If $f, g$ are scalar fields, show that $\nabla f \times \nabla g$ is solenoidal.

(b) Find the moment of inertia of a hallow sphere about a diameter. Its external and internal radii being 5 meters and 4 meters.

[6+10]

7.(a) Determine the natural frequencies and normal modes of vibrating system for which $M = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and stiffness $K = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$.

(b) Verify *Cayley-Hamiltion* theorem for $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$

[8+8]

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