

I B. Tech II Semester Supplementary Examinations, January/February - 2023

MATHEMATICS-III

(Common to all Branches)

Time: 3 hours

Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answering the question in **Part-A** is Compulsory
 3. Answer any **THREE** Questions from **Part-B**

PART -A (22 Marks)

1. a) Reduce to echelon form and hence find the rank of the matrix A [4M]

$$= \begin{bmatrix} 1 & 2 & 4 & 2 \\ 2 & 1 & 3 & 6 \\ 2 & 4 & 6 & 7 \end{bmatrix}.$$
- b) Find the Eigen values of $2A^3 - 3A^2 + 5A^{-1}$ if the Eigen values of A are 2, 3, & 4. [4M]
- c) Find the perimeter of the cardioid $r = a(1 + \cos \theta)$ [4M]
- d) Find $\Gamma\left(\frac{5}{2}\right)$ [3M]
- e) Find grad ϕ where $\phi(x, y, z) = e^{(x^2+y^2+z^2)}$ at (1,1,1) [3M]
- f) Evaluate $\int \vec{f} \cdot d\vec{r}$ where $\vec{f} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ along the path [4M]
 $x = 2t^2, y = t, z = t^3.$

PART -B (48 Marks)

2. a) Express the following system in matrix form and solve by Gauss elimination method. $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16.$ [8M]
- b) Solve the system of equations $x + y - 2z + 3w = 0; x - 2y + z - w = 0; 4x + y - 5z + 8w = 0; 5x - 7y + 2z - w = 0$ [8M]
3. a) Reduce the Q.F. $3x^2 + 2y^2 + z^2 + 4xy + 12yz + 8xz$ to the Canonical form hence find rank index and signature. [8M]
- b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$ [8M]
4. a) Evaluate $\iiint_V xyz \, dx \, dy \, dz$ where v is bounded by the co-ordinate planes and the plane $x + y + z = 1.$ [8M]
- b) Evaluate $\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} xy \, dx \, dy$ by changing the order of the integration. [8M]
5. a) Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^n}} dx.$ [8M]
- b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sec \theta} \, d\theta$ using Beta -Gamma relation. [8M]

6. a) Prove that $\text{Curl}(\text{grad } \phi) = \vec{0}$. [8M]
b) Find the directional derivative of $\phi = xy + yz + zx$ at A in the directional of \overline{AB} [8M]
where $A(1, 2, -1)$ and $B(1, 2, 3)$.
7. a) Evaluate $\int_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ where 's' is the surface of the [8M]
cylinder $x^2 + y^2 = 1$ in the first octant between $z = 0$ and $z = 2$.
- b) Evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ where c is the boundary of the [8M]
triangle with vertices $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$. using Stoke's theorem

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