Code No: R13202 **R13** **SET-1**

I B. Tech II Semester Supplementary Examinations, March - 2022
MATHEMATICS-III
(Common to all branches)

Time: 3 hours Max. Marks: 70

Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)
2. Answering the question in **Part-A** is Compulsory
3. Answer any **THREE** Questions from **Part-B**

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PART –A

1. a) Reduce the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ to its normal form and hence find the rank. (4M)

b) Prove that the matrix similar matrices have same Eigen values. (4M)

c) Find the length of the arc of the curve $y^2 = 4ax$ from $y = 0$ to 2. (4M)

d) Find grad φ if φ= $x^2+y^2+z^2=9$ at point (2,1,2). (3M)

e) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (3M)

f) Find $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = x^2y^2\,\bar{i} + y\,\bar{j}$ and the 'C' is the curve $y^2 = 4x$ in xy plane from (0, 0) to (4, 4). (4M)

PART -B

2. a) Solve the equations $x + y + z - w = 2, 7x + y + 3z + w = 12, 8x - y + z - 3w = 5, 10x + 5y + 3z + 2w = 20.$ by Gauss-Jordan method. (8M)

b) Determine whether the following equations will have a non-trivial Solution if so solve them x+y-2z+3w=0 ; x-2y+z-w=0 ; 4x+y-5z+8w=0 ; 5x-7y+2z-w=0 (8M)

3. a) Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2yz + 2zx - 2xy$ in to canonical form by using diagonal form hence find rank, index and signature. (8M)

b) Verify cayley Hamilton the matrix A= $\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ find A^{-1} (8M)

4. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (1 - x^2 - y^2)^{1/2}\, dx\, dy$ by changing into polar form. (8M)

b) Evaluate $\int_0^{\pi/2} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2-r^2}} r\, dr\, d\theta\, dz$ (8M)
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5. a) Prove that $curl\ grad\phi = 0$ (8M)

   b) Prove that $\nabla^2\left(\frac{x}{r^3}\right) = 0$ (8M)

6. a) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{\beta(m,n)}{(a+b)^m a^n}$ (8M)

   b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{7}{2}}\theta\cos^{\frac{2}{3}}\theta d\theta$ using Beta –gamma functions. (8M)

7. a) Evaluate $\oint_c yz\,dx + zx\,dy + yx\,dz$ where c is the curve $x^2 + y^2 = 1$, $z = y^2$ using stoke's theorem. (8M)

   b) Using Green's theorem evaluate $\oint_c e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ where c is the rectangle with vertices (0,0) , (π,0),(π,π/2) ,(0,π/2) . (8M)