

**I B. Tech II Semester Supplementary Examinations, April/May - 2019**  
**MATHEMATICS-III**  
 (Com. to all branches)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)  
 2. Answering the question in **Part-A** is Compulsory  
 3. Answer any **THREE** Questions from **Part-B**

**PART -A**

1. a) Find the Rank of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$  using Echelon form. (3M)
- b) If  $\lambda$  is an Eigen value of a non singular matrix A. Show that  $|A| / \lambda$  is an Eigen value of the adj A (3M)
- c) Evaluate  $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$  (4M)
- d) Prove that  $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$  (4M)
- e) Prove that  $\nabla \left( \frac{1}{r} \right) = \frac{-\vec{r}}{r^3}$  (4M)
- f) Find the work done in moving particle in the force field  $\vec{F} = 3x^2 \vec{i} + \vec{j} + z\vec{k}$  along the straight line (0, 0, 0) to (2, 1, 3) (4M)

**PART -B**

2. a) Solve the equations  $5x + y + z + w = 4, x + 7y + z + w = 12, x + y + 6z + w = -5, x + y + z + 4w = -6$ . by Gauss-seidal method. (8M)
- b) Test for consistency and solve  $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$ . (8M)
3. a) If  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$  Verify Cayley Hamilton theorem. Find  $A^4$  (8M)
- b) Find the Characteristic roots and Eigen vectors of the matrix  $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$  (8M)
4. a) Solve  $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} dx dy dz$  (8M)
- b) Evaluate  $\int_0^a \int_y^a \frac{x dx dy}{x^2 + y^2}$  by change of order of Integration. (8M)

5. a) Show that  $\beta(m, n) = a^m b^n \int_0^{\infty} \frac{x^{m-1}}{(ax+b)^{m+n}} dx$  (8M)
- b) Evaluate  $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$  (8M)
6. a) Find the directional derivative of the function  $e^{2x} \cos yz$  at the origin in the direction to the tangent to the curve  $x = a \sin t, y = a \cos t, z = at$  at  $t = \frac{\pi}{4}$  (8M)
- b) Prove that  $\text{curl grad} \phi = 0$ . (8M)
7. a) Evaluate  $\iint_s (\nabla \times \vec{F}) \cdot \vec{n} ds$  where  $\vec{F} = (x^2 + y - 4)\vec{i} + 3xy\vec{j} + (2xy + z^2)\vec{k}$  and  $s$  in the surface of the paraboloid  $z = 4 - x^2 - y^2$  above the  $xy$  plane using stoke's theorem. (8M)
- b) Verify Green's theorem for scalar line integral of  $f = (x^2 + y^2)\vec{i} - 2xy\vec{j}$  around the Rectangle determined by the lines  $x = \pm a; y = 0; y = b$ . (8M)