Code No: **R13202** **R13** **SET-1**

**I B. Tech II Semester Supplementary Examinations, April/May - 2018**

# MATHEMATICS-III

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **THREE** Questions from **Part-B**

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## PART –A

1. a) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form and hence find the rank. (4M)

b) Prove that $|A| / \lambda$ is an Eigen value of the matrix adj A. (3M)

c) Find the asymptotes in the curve $y^2(a+x) = x^2(3a-x)$. (3M)

d) Prove that $B(m,n) = B(m+1,n) + B(m,n+1)$. (4M)

e) Find the unit normal vector to the surface $\varphi(x,y,z) = x^2 + y^2 + z^2$ at $(-1,-1,-2)$ (4M)

f) Evaluate $\int f.dr$ where $f = (2xy + 3z^3)i + x^2 j + 3xz^2 k$ along the straight line joining (0,0,0) and (2,1,2). (4M)

## PART -B

2. a) Show that the only real value of λ for which the following equations have non-trivial solution is 6 and solve them, when λ=6. x+2y+3z=λx; 3x+y+2=λy; 2x+3y+z=λx. (8M)

b) Solve the equations $5x + y + z + w = 4, x + 7y + z + w = 12, x + y + 6z + w = -5, x + y + z + 4w = -6$ by Gauss-seidal method. (8M)

3. a) Reduce the quadratic form $x^2+y^2+2z^2-2xy+4xz+4yz$ to the canonical form and find the rank , index and signature. (8M)

b) Verify cayley Hamilton for the matrix $A=\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ find $A^{-1}$ (8M)
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4. a) Find the perimeter of the Loop of the curve $3ay^2=x(x-a)^2$. (8M)

b) Change the order of the Integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the double integral. (8M)

5. a) Prove that $\Gamma(n)\Gamma(1-n)=\dfrac{\pi}{\sin n\pi}$ (8M)

b) Prove that $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^n}}=\frac{\sqrt{\pi}}{n}\frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$ (8M)

6. a) Find the directional derivative of $\varphi(x,y,z)=x^4+y^4+z^4$ at the point (-1,2,3) in the direction towards the point (2,-1,-1). (8M)

b) If $\varphi$ be two scalar point functions and $\overline{f}$ be two vector point functions then show that $\nabla.(\phi\overline{f})=\nabla\phi\,.\overline{f}+\phi(\nabla.\overline{f})$ (8M)

7. a) Verify Gauss divergence theorem $F=(x^3-yz)i-2x^2yj+zk$ over the surface of the cube bounded by x = y = z = a. (8M)

b) Evaluate $\int_c (xy-y^2)dx+x^2ydy$ along the closed curve formed by y = 0, x =1 and y = x by greens theorem. (8M)