Subject Code: R13202/R13 **Set No - 1**

I B. Tech II Semester Regular/Supply Examinations July - 2015

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(a) Find for what values of 'a' the equations, x + y + z =1, x + 2y + 4z =a and x + 4y + 10z= a^2 have a solution.

(b) Find the moment of inertia about the initial line of the cardioids r = a(1+cosθ).

(c) What is the nature of the quadratic form X^TAX, if A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(d) Evaluate $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}$.

(e) If ∅ satisfies Laplace equation, show that $\nabla\phi$ is both solenoidal and irrotational.

(f) Use Greens theorem to evaluate $\int_c (2xy - x^2)dx + (x^2 + y^2)dy$, where c is the closed curve of the region bounded by $y=x^2$ and $y^2=x$.

[3+4+4+4+3+4]

PART-B

2.(a) Solve the system of equations $8x - 3y + 2z = 20, 4x + 11y - z = 33$ and $6x + 3y + 12z = 36$ using Gauss-Seidel method.

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

A= $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

[8+8]

3.(a) Find Eigen values and Eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b) Reduce the Quadratic form to canonical form by orthogonal reduction and state the nature of the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$.

[8+8]

4.(a) Find surface area of the right circular cone generated by the revolution of right angled triangle about a side which contains a right angle.

(b) Evaluate $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} y\,dx\,dy$ by changing the order of integration.

[8+8]

5.(a) Evaluate $\int_0^\infty x^2 e^{-x^2} dx$.

(b) Express $\int_0^\infty \frac{x^c}{c^x} dx$, $(c > 1)$ in terms of Gamma function.

[8+8]

6.(a) Find the directional derivative of the function $\phi = xy^2 + yz^3$ at (2,-1,1) in the direction of normal to the surface $x \log z - y^2 + 4 = 0$ at (-1,2,1).

(b) Show that $\frac{\bar{r}}{r^3}$ is solenoidal, where $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$.

[8+8]

7.(a) If $\bar{F} = xy\,\bar{i} - z\,\bar{j} + x^2\,\bar{k}$ and C is the curve $x = t^2, y = 2t$, and $z = t^3$ from t=0 to t=1, find the work done by $\bar{F}$.

(b) Use divergence theorem to evaluate $\iint_S \bar{F} \cdot dS$ where $\bar{F} = 4x\,\bar{i} - 2y^2\,\bar{j} + z^2\,\bar{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4$, z=0 and z=3.

[8+8]

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Subject Code: R13202/R13 **Set No - 2**

I B. Tech II Semester Regular/Supply Examinations July - 2015

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(a) Find the value of 'a' for which the system of equations 3x-y+ 4z =3; x+2y -3z =-2 and 6x+5y +az = - 3 will have infinite number of solutions.

(b) If 2, 3, 5 are the eigenvalues of matrix A, then find the eigenvalues of $2A^3+3A^2+5A+3I$.

(c) Find the moment of inertia about the initial line of the cardioid r = a(1- cosθ).

(d) Evaluate $\int_0^1 \frac{x^3}{\sqrt{1-x^5}}dx$ in terms of Beta functions.

(e) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1, 2).

(f) Evaluate $\int_c (e^x dx+2y dy-dz)$ where c is the curve is the curve $x^2+y^2=9$, z=2, by using Stoke's theorem.

[3+3+4+4+4+4]

PART- B

2.(a) Solve the equations $3x+y+2z=3, 2x-3y-z=-3$ and $x+2y+z=4$ using Gauss elimination method.

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A=\begin{bmatrix}2 & -2 & 0 & 6\\4 & 2 & 0 & 2\\1 & -1 & 0 & 3\\1 & -2 & 1 & 2\end{bmatrix}.$$

[8+8]

3.(a) Find A^{-1} using Cayley-Hamilton theorem , where $A=\begin{bmatrix}1 & 2 & 3\\2 & 4 & 5\\3 & 5 & 6\end{bmatrix}$.

(b) Reduce the Quadratic form $x^2+3y^2+3z^2-2yz$ into canonical form and find the nature, rank, index and signature.

[8+8]

4.(a) Find the surface area generated by the revolution of an arc of the catenary $y = c\cosh\frac{x}{c}$ about x-axis.

(b) Change the order of integration and evaluate $\int_0^a \int_x^a \left(x^2 + y^2\right) dy dx$.

[8+8]

5.(a) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$

(b) Express $\int_0^1 x^m (1-x^n)^p$ in terms of Γ function.

[8+8]

6.(a) Find the directional derivative of $\frac{1}{r}$ in the direction of $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ at (1,1,2).

(b) If $\bar{A}$ is irrotational, evaluate $div\left(\bar{A} \times \bar{r}\right)$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$.

[8+8]

7.(a) Find the work done by the force $z\bar{i} + x\bar{j} + y\bar{k}$, if it moves a particle along the arc of the curve $\bar{r} = \cos t\bar{i} + \sin t\bar{j} - t\bar{k}$ from $t = 0$ to 2π .

(b) Compute $\int (ax^2 + by^2 + cz^2) ds$ over the surface of the sphere $x^2 + y^2 + z^2 = 1$.

[8+8]

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Subject Code: R13202/R13 **Set No - 3**

I B. Tech II Semester Regular/Supply Examinations July - 2015

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(a) Find for what values of 'a' such that the rank of the matrix A is 2, where

$$A=\begin{bmatrix}1 & 1 & -1 & 1\\ 1 & -1 & a & -1\\ 3 & 1 & 0 & 1\end{bmatrix}.$$

(b) Prove that the Eigen values of a Skew-Hermitian matrix are either purely imaginary or zero.

(c) Find the length of the curve $3x^2 = y^3$ between y=0 and y=1.

(d) Evaluate $\int_0^1 \frac{dx}{(1-x^3)^{1/3}}$ using Beta functions.

(e) Find div $\overline{F}$, where $\overline{F} = r^n\,\overline{r}$. Find n if it is solenoidal.

(f) Using Stoke's theorem, evaluate the integral $\int_c \overline{F}\cdot dr$, where

$\overline{F} = 2y^2\overline{i} + 3x^2\overline{j} - (2x+z)\overline{k}$ and c is the boundary of the triangle whose vertices are *(0,0,0)* ,*(2,0,0)* and *(2,2,0).*

[3+3+4+4+4+4]

PART- B

2.(a) Using Gauss-Jordon method solve the system of equations
$2x + y + z = 10, 3x + 2y + 3z = 18$ and $x + 4y + 9z = 16$.

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A=\begin{bmatrix}2 & 1 & 3 & 4\\ 0 & 3 & 4 & 1\\ 2 & 3 & 7 & 5\\ 2 & 5 & 11 & 6\end{bmatrix}.$$

[8+8]

3.(a) Find A^{-1} by using Cayley-Hamilton theorem, where $A=\begin{bmatrix}1 & 2 & -1\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$.

(b) Reduce the Quadratic form to canonical form by orthogonal reduction and state the nature of the quadratic form $2x^2+2y^2+2z^2-2xy-2yz-2zx$.

[8+8]

4.(a) Find the volume obtained by revolving the loop of the curve $x=t^3, y=t-\frac{t^3}{3}$ about x-axis.

(b) Change the order of integration and evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx$.

[8+8]

5.(a) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}dx$ using β and Γ function.

(b) Show that $\int_0^\infty x^m e^{-ax^n} dx = \frac{1}{na^{\frac{m+1}{n}}}\Gamma\left(\frac{m+1}{n}\right)$.

[8+8]

6.(a) Find the directional derivative of $x^2-2y^2+4z^2$ at (1,1,-1) in the direction of $2\bar{i}+\bar{j}-\bar{k}$

(b) Find a,b,c so that $\bar{A}=(x+2y+az)\bar{i}+(bx-3y-z)\bar{j}+(4x+y+2z)\bar{k}$ is irrotational. Also find ϕ such that $\bar{A}=\nabla\phi$.

[8+8]

7.(a) Compute the line integral $\int(y^2dx-x^2dy)$ round the triangle whose vertices are *(1,0), (0,1)* and *(-1,0).*

(b) Use divergence theorem to evaluate $\iint_S \bar{F}\cdot d\bar{S}$ where $\bar{F}=x^3i+y^3j+z^3k$ and S is surface of the sphere $x^2+y^2+z^2=r^2$.

[8+8]

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Subject Code: R13202/R13

Set No - 4

I B. Tech II Semester Regular/Supply Examinations July - 2015

MATHEMATICS-III

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART-A

1.(a) Find the values of 'a' and 'b' for which equation
$x + y + z = 3; x + 2y + 2z = 6; x + ay + 3z = b$ have unique solution.

(b) Prove that the eigenvalues of a Skew-Hermitian matrix are either purely imaginary or zero.

(c) Find the moment of inertia about the initial line of the cardioid r = a(1+cosθ).

(d) Evaluate $\int_0^1 \frac{x^3}{\sqrt{1-x^5}}\,dx$ in terms of Beta functions.

(e) Find the directional derivative of $2xy + z^2$ at (1, -1, 3) in the direction of $\bar{i} + 2\bar{j} + 3\bar{k}$

(f) Using Stoke's theorem, evaluate $\int_c \overline{F}\cdot dr$, where $\overline{F} = 2y^2\bar{i} + 3x^2\bar{j} - (2x+z)\bar{k}$ and c is the boundary of the triangle whose vertices are *(0,0,0)* ,*(1,0,0)* and *(1,1,0)*.

[3+3+4+4+4+4]

PART- B

2.(a) Solve the system of equations using Gauss-Seidel method correct to three decimal place $8x - 3y + 2z = 20, 4x + 11y - z = 33$ and $6x + 3y + 12z = 36$.

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}.$$

[8+8]

3.(a) Find Eigen values and Eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b) Reduce the Quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$ into canonical form and find the nature, rank, index and signature.

[8+8]

4.(a) Find the volume of the solid of revolution generated by the revolution of the cissoid $y^2 = \frac{x^3}{2a - x}$ about its asymptote.

(b) Change the order of integration and evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy^2 dydx$.

[8+8]

5.(a) Evaluate $\int_5^7 (x-5)^6 (7-x)^3 dx$ using β and Γ functions.

(b) Express $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$ in terms of Γ function.

[8+8]

6.(a) If Prove that $\nabla \cdot \left[r\nabla\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}$, where $r = \sqrt{x^2 + y^2 + z^2}$.

(b) Find the directional derivative of $x^2 - 2y^2 + 4z^2$ at (1, 1,-1) in the direction of $2\bar{i} + \bar{j} - \bar{k}$.

[8+8]

7.(a) If $\overline{F} = (5xy - 6x^2)\bar{i} + (2y - 4z)\bar{j}$, evaluate $\int_c \overline{F} \cdot dr$ along the curve c: $y=x^3$ from (1,1) to (2,8).

(b) Apply Stoke's theorem to evaluate $\oint_c (ydx + zdy + xdz)$ where c is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x+z = a$.

[8+8]

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