Code No: R13202 **R13** **SET-1**

I B. Tech II Semester Supplementary Examinations, July/August- 2021

MATHEMATICS-III

(Com. to all branches)

Time: 3 hours Max. Marks: 70

Note: 1. Question paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **THREE** Questions from **Part-B**

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PART –A

1. a) Prove that if one Eigen value of a matrix is '0', then A is symmetric matrix. (4M)

b) If the Eigen values of the matrix corresponding to a quadratic form are -3, -3 and 5, then what are the index, signature and nature of the quadratic form? 3M)

c) Evaluate $\int_0^2 (8-x^3)^{1/3}\,dx$ using β and γ functions. (4M)

d) Evaluate $\int_0^1 dx\int_0^x e^{y/x}dy$. (4M)

e) If $\phi = ax^2 + by^2 + cz^2$ satisfies Laplacian equation, then what is the value of a + b + c? (4M)

f) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then evaluate $\oint_c \bar{r}.d\bar{r}$. (3M)

PART -B

2. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$ (8M)

b) Apply Gauss-Seidel method to solve the equations *2x+y+6z=9; 8x+3y+2z=13; x+5y+z=7.* (8M)

3. a) Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. (8M)

b) Using Cayley-Hamilton theorem, find A^{-1}, where A = $\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (8M)

4. a) Prove that the length of the arc of a loop of the curve $9ay^2 = x(x-3a)^2$ is $4\sqrt{3}a$. (8M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$ by changing into polar coordinates. (8M)
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5. a) Evaluate $\int_0^{\infty} \frac{x}{1+x^4} dx$ using β and γ functions. (8M)

b) $Evaluate \int_0^1 x(1-x^3)^{1/3} dx$ (8M)

6. a) Find the angle between the normals to the surface $x^2 = yz$ at the points (1, 1, 1) and (2, 4, 1). (8M)

b) Prove that curl $(\bar{a} \times \bar{b}) = \bar{a} div \bar{b} - \bar{b} div \bar{a} + (\bar{b}.\nabla)\bar{a} - (\bar{a}.\nabla)\bar{b}$. (8M)

7. a) Evaluate $\int_C (xy + y^2)dx + (x^2)dy$ where C is the curve bounded y = x^2 and y = x. (8M)

b) Evaluate $\iint_S \bar{F}.\, \bar{n}\ ds$ if $\bar{F} = yz\bar{i} + 2y^2\bar{j} + xz^2\bar{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between the planes z = 0 and z = 2. (8M)