Code No: R13202 **R13** **SET-1**

I B. Tech II Semester Supplementary Examinations, Nov/Dec - 2018

MATHEMATICS-III

(Com. to all branches)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **THREE** Questions from **Part-B**

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PART –A

1. a) Reduce to echelon form and hence find the rank of the matrix (4M)

$$A = \begin{bmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{bmatrix}$$

b) Prove that the sum of the given values of a matrix is the trace of A and product of the Eigen values of A is the determinant of A. (4M)

c) Find the perimeter of the cardiod $r = a(1 - \cos\theta)$ (4M)

d) Find $\Gamma\left(\frac{1}{2}\right)$ (3M)

e) Find grad ϕ where $\phi(x, y, z) = \log\left(x^2 + y^2 + z^2\right)$ at $(1,1,1)$ (3M)

f) Find work done in moving particle in the force field $\bar{F} = 3x^2\,\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the space curve $x = 2t^3, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$. (4M)

PART –B

2. a) Express the following system in matrix form and solve by Gauss elimination method. (8M)

$2x_1 + x_2 + 2x_3 + x_4 = 6$;
$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$;
$4x_1 + 3x_2 + 3x_3 - 3x_4 = 1$;
$2x_1 + 2x_2 - x_3 + x_4 = 10$.

b) Show that the system of equations : (8M)

$2x_1 - 2x_2 + x_3 = \lambda x_1$;
$2x_1 - 3x_2 + 2x_3 = \lambda x_2$;
$-x_1 + 2x_2 = \lambda x_3$

can possess a non-trivial solution only if $\lambda = 1$ $\lambda = -3$ and obtain the general solution in each case.

3. a) Reduce the Q.F. $3x^2 - 2y^2 - z^2 - 4xy + 12yz + 8xz$ to the Canonical form (8M)

b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (8M)
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4\. a) Evaluate $\iint x^3 y\, dx\, dy$ over the region enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant. (8M)

b) Evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy\, dx\, dy$ by changing the order of the integration. (8M)

5\. a) Show that (8M)

(i) $\Gamma(x)\Gamma(-x)=\dfrac{-\pi}{x\sin\pi x}$

(ii) $\Gamma\left(\dfrac{1}{2}+x\right)\Gamma\left(\dfrac{1}{2}-x\right)=\pi Sec\pi x$

b) Show that $\beta(m,n)=\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$ and deduce that (8M)

$$\int_0^{\frac{\pi}{2}} \sin^n\theta d\theta=\int_0^{\frac{\pi}{2}} \cos^n\theta d\theta=\frac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{n+2}{2}\right)}$$

6\. a) Find the values of a, b, c if the directional derivative of the function $\phi=axy^2+byz+cz^2x^3$ at the (1,2,-1) has the maximum magnitude 64 in the direction parallel to z axis . (8M)

b) Prove that $\nabla^2(\log r)=\dfrac{2}{r^2}$ (8M)

7\. a) Verify stoke's theorem for $\bar{F}=x^2\,\bar{i}+xy\,\bar{j}$ around the square in $z=0$ plane whose sides are along the lines $x=0; y=0$; $x=1$, $y=1$. (8M)

b) Evaluate $\oint_c \sin y\,dx + x(1+\cos y)\,dy$ by using green's theorem over the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $z=0$. (8M)