I B. Tech II Semester Supplementary Examinations, November - 2021

MATHEMATICS-III

(Com. to all branches)

Time: 3 hours Max. Marks: 70

Note: 1. Question paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **THREE** Questions from **Part-B**

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PART –A

1. a) Prove that if λ is an eigen values of a matrix A then $1/\lambda$ is an Eigen value of A^{-1}. (4M)

b) If the eigen values of the matrix corresponding to a quadratic form are 2, -3 and 5, then what are the index, signature and nature of the quadratic form? (3M)

c) Evaluate $\int_0^1\int_0^x e^x\,dxdy.$ (4M)

d) Evaluate $\int_0^{\pi/2}\sin^{11/2}x\,dx.$ (3M)

e) If $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$ then find the $\nabla^2\left(\frac{1}{r}\right)$. (4M)

f) Evaluate, if $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$, $\int_S \bar{r}\times\bar{n}\,dS$. (4M)

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PART -B

2. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix}0&1&-3&-1\\1&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$ (8M)

b) Apply Gauss-Jacobi method to solve the equations $27x+6y-z=85;\ x+y+54z=40;\ 6x+15y+2z=72.$ (8M)

3. a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix}1&1&1\\-1&-3&-3\\2&4&4\end{bmatrix}$. (8M)

b) Using Cayley-Hamilton theorem, find A^{-1}, where $A=\begin{bmatrix}1&2&-1\\2&1&-2\\2&-2&1\end{bmatrix}$. (8M)

4. a) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$. (8M)

b) Evaluate $\iint(x+y)\,dxdy$, over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ (8M)
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5. a) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx$ in terms of Gamma function. (8M)

b) Evaluate $\int_0^{\pi/2} \sin^{5/2}\cos^{3/2} dx$. (8M)

6. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$. (8M)

b) Prove that $\nabla \times \left(\frac{\bar{A}\times\bar{r}}{r^n}\right) = \frac{(2-n)\bar{A}}{r^n} + \frac{n(\bar{r}.\bar{A})\bar{r}}{r^{n+2}}$. (8M)

7. a) If $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$, evaluate $\int_S \bar{F}.\bar{n}ds$ where S is the surface of the cube bounded by x = 0, x = a, y = 0, y = a, z = 0, z = a. (8M)

b) Evaluate $\int_c (x^2 + xy)dx + (x^2 + y^2)dy$, where C is the square formed by the lines x= 1 ,x = -1, y = 1, y = -1. (8M)