Subject Code: R13202/R13 **Set No - 1**

I B. Tech II Semester Supplementary Examinations Dec - 2016
MATHEMATICS-III
(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART A

1. a) Reduce the matrix $\begin{pmatrix} -1 & 2 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 1 \end{pmatrix}$ into Echelon form and hence find its rank.

   b) If λ is an eigenvalue of a non-singular matrix A, then show that $\frac{|A|}{\lambda}$ is an eigenvalue of the matrix $adjA$.

   c) Evaluate $\int_0^1 \int_y^1 \int_0^{1-x} xdzdydx$.

   d) Prove that $\int_0^\infty e^{-x^4} dx = \frac{1}{4}\Gamma\left(\frac{1}{4}\right)$

   e) Find $curl\overline{F}$ where $\overline{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$.

   f) Find the work done in moving a particle in the force field $\overline{F} = 3x^2\bar{i} + \bar{j} + z\bar{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$. (3M+4M+4M+4M+3M+4M)

PART B

2. a) Find the rank of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ after reducing it to echelon form.

   b) Use Gauss Seidel method to solve 25x + 2y + 2z = 69, 2x + 10y + z = 63, x + y +z = 43.

(8M+8M)

3. a) Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 2 & -1\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and find A^{-1}.

   b) Reduce the quadratic form $6x_1^2+3x_2^2+14x_3^2+4x_1x_2+18x_3x_1+4x_3x_2$ to canonical form. Also find signature and rank of the quadratic form. (8M+8M)

4. a) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y=8x$.

   b) Evaluate $\int_0^1\int_0^{\sqrt{1+x^2}} \frac{dydx.}{1+x^2+y^2}$ (8M+8M)

5. a) Evaluate $\int_0^1 x^4(\log\frac{1}{x})^3 dx$ using Beta and Gamma functions

   b) Show that $\int_0^\infty e^{-y^{1/m}} dy = m\Gamma m$ (8M+8M)

6. a) Find the directional derivative of $\phi = x^2yz+4xz^2$ at (1, -2, -1) in the direction of $2\bar{i}-\bar{j}-2\bar{k}$.

   b) Prove that $\nabla\times\left(\frac{\bar{A}\times\bar{r}}{r^n}\right)=\frac{(2-n)\bar{A}}{r^n}+\frac{n(\bar{r}.\bar{A})\bar{r}}{r^{n+2}}$. (8M+8M)

7. a) Find the work done by $\bar{F}=(2x-y-z)\bar{i}+(x+y+z)\bar{j}+(3x-2y-5z)\bar{k}$ along a curve C in the xy – plane given by $x^2+y^2=9$, z = 0.

   b) Evaluate $\iint_S \bar{F}.\bar{n}\,ds$ if $\bar{F}=yz\bar{i}+2y^2\bar{j}+xz^2\bar{k}$ and S is the surface of the cylinder $x^2+y^2=9$ contained in the first octant between the planes z = 0 and z = 2. (8M+8M)