

Subject Code: R13207/R13

Set No - 1

I B. Tech II Semester Supplementary Examinations Feb. - 2015

**MATHEMATICS-II (MATHEMATICAL METHODS)**

(Common to CE, ME, CSE, PCE, IT, Chem E, Aero E, Auto E, Min E, Pet E, Metal E)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B**  
Answering the question in **Part-A** is Compulsory,  
Three Questions should be answered from **Part-B**

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**PART-A**

- 1.(i) Solve  $x^3 - 2x - 5 = 0$  for a positive root by iteration method.
- (ii) Express shift operator E in terms of exponential function.
- (iii) Using Euler's method, solve for y at  $x = 0.2$  from  $\frac{dy}{dx} = y^2 + x$ ,  $y(0) = 1$ , taking step size  $h = 0.1$ .
- (iv) Find  $b_n$ , Using the Fourier series of the function  $f(x)$  is given by
$$f(x) = \begin{cases} \frac{1}{2} + x, & \text{for } -1 \leq x \leq 0 \\ \frac{1}{2} - x, & \text{for } 0 \leq x \leq 1 \end{cases}$$
- (v) State Fourier integral theorem and then obtain Fourier integral in complex form.
- (vi) Using  $Z[2.3^n + 5.n] = \frac{2z}{z-3} + \frac{5z}{(z-1)^2}$ , find  $Z[2.3^{n+3} + 5(n+3)]$  using shifting theorem.

[4+3+4+3+4+4]

**PART-B**

- 2.(a) Using Regula-Falsi Method, find the real root of,  $x^3 - x - 4 = 0$ .
- (b) Construct the difference table for the following data

x	0.2	0.4	0.6	0.8	1.0
y	0.001	0.0045	0.12	0.25	0.45

and evaluate  $f(0.7)$ .

[8+8]
- 3.(a) Prove that (i)  $\nabla E = E \nabla = \Delta$  (ii)  $\delta E^{1/2} = \Delta$ .
- (b) Find the root of  $x \sin x + \cos x = 0$  using Newton-Raphson method.

[8+8]
- 4.(a) Evaluate  $y(0.2)$  and  $y(0.4)$  correct to three decimals by Taylor's method if  $y(x)$  satisfies  $y' = 1 - 2xy$ ,  $y(0) = 0$ .
- (b) Find the Fourier series of the function  $f(x) = \begin{cases} 0, & 0 < x < 1 \\ x^2, & 1 < x < 2 \end{cases}$ .

[8+8]

5.(a) Find  $y(0.1)$  and  $y(0.2)$  using Runge-Kutta 4<sup>th</sup> order formula, given that  $y' = x^2 - y$ ,  $y(0)=1$ .

(b) Express  $f(x) = x$  as a half range cosine series in  $1 < x < 2$  and hence prove that

$$\frac{8}{\pi^2} \left[ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots \right] = 1.$$

[8+8]

6.(a) Find the Fourier transform of  $e^{-x^2}$

(b) Find  $Z[\cosh at \cdot \sin bt]$

[8+8]

7.(a) Find the finite Fourier transform of  $f(x) = x^3$  in  $(0, \pi)$ .

(b) Solve the difference equation  $u_{n+2} - 3u_{n+1} + 2u_n = 0, u_0 = 0, u_1 = 1$  using Z-transform.

[8+8]

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Set No - 2

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**PART-A**

- 1.(i) Find a real root of the equation  $2x - \log x = 7$  using iteration method.
- (ii) Show that  $e^x \left( u_0 + x\Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \dots \right) = u_0 + u_1 x + u_2 x^2 + \dots$
- (iii) Solve  $\frac{dy}{dx} = 1 + xy$ ,  $y(0) = 1$  by using Picard's method up to 2 approximations.
- (iv) Find  $a_n$ , using the Fourier series of the function  $f(x)$  given by
- $$f(x) = \begin{cases} -k, & \text{for } -\pi < x < 0 \\ k, & \text{for } 0 < x < \pi \end{cases}$$
- (v) Prove that  $F_s\{f'(x)\} = -pF_c(p)$ .
- (vi) If  $f(z) = \frac{2z^2 + 4z + 12}{(z-1)^4}$ , find the value of  $f(2)$ .

[4+4+4+3+3+4]

**PART-B**

- 2.(a) Using Regula-Falsi method, find the root of  $x^3 - x - 2 = 0$  over (1, 2).
- (b) Find  $f(2.5)$  using Newton's forward formula for the following table

x	0	1	2	3	4	5
y	0	1	8	21	72	94

[8+8]

- 3.(a) Find the Lagrange's interpolation formula for the data

X	0	2	5	6
f(x)	2	4	12	14

and hence evaluate  $f(3.5)$ .

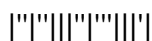
- (b) By using Newton-Raphson method, find the root of  $x^4 - x - 10 = 0$  correct to three decimal places.

[8+8]

- 4.(a) Find by Taylor's series method the value of  $y$  at  $x = 0.1$  up to 3 decimal places, given that  $y' = x^2 y - 1$ ,  $y(0) = 1$ .

- (b) Obtain the Fourier series of  $f(x) = e^x$  in the range  $0 < x < 2\pi$ .

[8+8]



5.(a) Given  $y' = x + \sin y$ ,  $y(0) = 1$ , compute  $y(0.2)$  and  $y(0.4)$  using Euler's modified method.

(b) Find Fourier cosine series of the function  $f(x) = \sin x$  in  $(0, \pi)$  and hence show that

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}.$$

[8+8]

6.(a) Express  $f(x) = \begin{cases} 1, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$  as Fourier sine integral and hence evaluate  $\int_1^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda \, d\lambda$

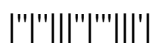
(b) Solve the difference equation  $y_{n+2} - 2y_{n+1} + y_n = 2^n$ ,  $y_0 = 2$ ,  $y_1 = 1$  using Z-transform.

[8+8]

7.(a) Find Fourier sine transform of  $e^{-|x|}$  and hence evaluate  $\int_0^{\infty} \frac{x \sin mx}{1 + x^2} dx$ .

(b) Find  $Z^{-1} \left[ \frac{8z - z^3}{(4 - z)^3} \right]$ .

[8+8]



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## MATHEMATICS-II (MATHEMATICAL METHODS)

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\*\*\*\*\*

**PART-A**

- 1.(i) Find a positive root of the equation by iteration method:  $3x = \cos x + 1$ .
- (ii) Using the method of separation of symbols, show that
 
$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}.$$
- (iii) Using Euler's method, solve for  $y(2)$  from  $\frac{dy}{dx} = 3x^2 + 1$ ,  $y(1) = 2$ , taking step size  $h=0.5$ .
- (iv) Using the Fourier series, Find  $b_n$  of the function  $f(x)$  given by
 
$$f(x) = \begin{cases} -x, & \text{for } -\pi < x < 0 \\ x, & \text{for } 0 < x < \pi \end{cases}.$$
- (v) If  $F_s(p)$  is the Fourier sine transform of  $f(x)$ , then show that
 
$$F_s[f(x) \cos ax] = \frac{1}{2} [F_s(p+a) + F_s(p-a)].$$
- (vi) If  $f(z) = \frac{5z^2 + 3z + 12}{(z-1)^4}$ , find the value of  $f(2)$ .

[3+4+4+3+4+4]

**PART-B**

- 2.(a) Using Regula-Falsi Method, find the root of  $xe^x - 2 = 0$ .
- (b) Use Gauss backward interpolation formula to find  $f(32)$  given that  $f(25) = 0.2707$ ,  $f(30) = 0.3027$ ,  $f(35) = 0.3386$ ,  $f(40) = 0.3794$ .  
[8+8]
- 3.(a) Find a real root of  $xe^x - \cos x = 0$  using Newton-Raphson method.(up to 4 decimals)
- (b) Use Lagrange's formula to fit a polynomial to the data
 

x	0	1	3	4
y	-12	0	6	12

 Also find  $y(2)$ .  
[8+8]
- 4.(a) Solve  $y' = y - x^2$ ,  $y(0) = 1$  using Picard's method up to 4<sup>th</sup> approx.
- (b) Obtain Fourier series of  $f(x) = x \sin x$ ,  $0 < x < \pi$  and Show that
 
$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{\pi - 2}{4}.$$

[8+8]

5.(a) Given  $y' = -xy^2$ ,  $y(0) = 2$ . Compute  $y(0.2)$  in steps of 0.1 using modified Euler's method.

(b) Expand  $\cos \pi x$  in  $(0, 1)$  as Fourier sine series.

[8+8]

6.(a) Solve the difference equation  $y_{n+2} + 2y_{n+1} + y_n = n$ ,  $y_0 = y_1 = 0$  using Z-transform.

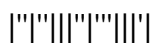
(b) Prove that  $F\left[\frac{d^n}{dx^n} F(x)\right] = (-ip)^n F(p)$  where  $F[f(x)] = F(p)$ .

[8+8]

7.(a) Using Fourier integral Show that  $e^{-x} \cos x = \frac{2}{\pi} \int_0^\infty \frac{\lambda^2 + 2}{\lambda^4 + 4} \cos \lambda x d\lambda$ .

(b) Find  $Z^{-1}\left[\frac{z^2 - 3z}{(z+2)(z-5)}\right]$ .

[8+8]



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\*\*\*\*\*

**PART-A**

- 1.(i) Solve  $x = 1 + \tan^{-1} x$  by iteration method.
- (ii) If the interval of differencing is unity, prove that  $\Delta \tan^{-1}\left(\frac{n-1}{n}\right) = \tan^{-1}\left(\frac{1}{2n^2}\right)$ .
- (iii) Solve  $\frac{dy}{dx} = 2x - y$ ,  $y(1) = 3$  by using Picard's method up to 2 approximations.
- (iv) Find  $a_n$ , Using the Fourier series of the function  $f(x)$  is given by
$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x), & \text{for } -\pi < x \leq 0 \\ \frac{1}{2}(\pi - x), & \text{for } 0 \leq x < \pi \end{cases}$$
- (v) State and prove Modulation theorem in Fourier transform.
- (vi) Using  $Z(n^2) = \frac{z^2 + z}{(z-1)^3}$ , find  $Z[(n+1)^2]$  using shifting theorem.

[4+3+4+3+4+4]

**PART-B**

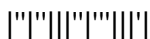
- 2.(a) Using Regula-Falsi Method, find the root of  $xe^x - 2 = 0$ .
- (b) Prove that  $(1 + \Delta)(1 - \nabla) = 1$ .

[8+8]
- 3.(a) Find the root  $x^3 - x - 4 = 0$  up to 3 decimal places using Newton-Raphson method.
- (b) Given that  $y(3) = 6$ ,  $y(5) = 24$ ,  $y(7) = 58$ ,  $y(9) = 108$ ,  $y(11) = 174$ , find  $x$  when  $y = 100$  using Lagrange's formula.

[8+8]
- 4.(a) Using Taylor's series method, solve  $y' = xy + y^2$ ,  $y(0) = 1$ ,  $x = 0.1$  and  $0.2$
- (b) Find Half range Fourier cosine series of  $f(x) = \cos ax$  ( $a$  is not an integer) in  $0 < x < \pi$ .

[8+8]
- 5.(a) Find the half-range cosine series for  $f(x) = x(2-x)$ , in  $0 \leq x \leq 2$  and hence find sum of the series  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
- (b) Given  $y' = x - y$ ,  $y(1) = 0.4$ , find  $y(1.2)$  using Runge-Kutta method.

[8+8]



6.(a) Find the inverse Fourier transform  $f(x)$  of  $F_s(p) = \frac{p}{1+p^2}$ .

(b) Solve the difference equation  $y_{n+2} - 6y_{n+1} + 8y_n = 2^n$ , using Z-transform.

[8+8]

7.(a) Using Fourier integral, Show that  $\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + a^2} dx = \frac{\pi}{2a} e^{-ax}, a > 0, x \geq 0$ .

(b) If  $\frac{3z^2 - 4z + 7}{(z-1)^3}$  is  $Z[f(n)]$ , then find  $f(0)$ ,  $f(1)$ , and  $f(2)$ .

[8+8]

