

I B. Tech II Semester Supplementary Examinations, March- 2022

MATHEMATICS-III

(Com to AE,AME,CE,CSE,IT,EIE,EEE,ME,ECE, Metal E, Min E, E Com E, Agri E, Chem. E, PCE,PE)
Time: 3 hours Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Find the value of k for which the rank of matrix $A = \begin{bmatrix} k & -1 & 0 \\ 0 & k & -1 \\ -1 & 0 & k \end{bmatrix}$ is '2'. (2M)
- b) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then find the eigenvalues of $2A^3 + 5A^2 - 3A + 5I$. (2M)
- c) Evaluate $\int_0^1 \int_0^1 (x + y) dx dy$ (2M)
- d) Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$ using Beta and Gamma functions. (2M)
- e) If $\phi = x^2 + y^2 + z^2 - 3xyz$ then find curl (grad ϕ). (2M)
- f) Find the angle between the sphere $x^2 + y^2 + z^2 = 9$ and the vector $4i - 2j - k$ at the point $(2, -1, 2)$ (2M)
- g) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then evaluate $\oint_c \vec{r} \cdot d\vec{r}$. (2M)

PART -B

2. a) Find rank of matrix $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ using Echelon form (6M)
- b) Solve the system of equations by Gauss -elimination method (8M)
 $2x - y + 2z + 6w = 4, 6x + y + 6z + 12w = 2, 4x + y + 3z - 3w = -1, 2x + 2y - z + w = 1$
3. a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix} 6 & 3 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (6M)

- b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_1x_2 + 18x_3x_1 + 4x_3x_2$ to canonical form using orthogonal transformation. Also find signature and rank of the quadratic form. (8M)
4. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2 y^2 (x+y) dy dx$. (6M)
- b) Using spherical polar coordinates, evaluate $\iiint xyz dx dy dz$ taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. (8M)
5. a) Show that $B(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$. (7M)
- b) Evaluate $\int_{-\infty}^{\infty} e^{-a^2 x^2} dx$, (7M)
6. a) Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$. (6M)
- b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - xz)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and also find its scalar potential. (8M)
7. a) Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the line joining $(0, 0, 0)$ to $(2, 1, 3)$. (7M)
- b) Using Green's theorem evaluate $\int_C (2xy - x^2) dx + (x^2 + y^2) dy$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$. (7M)