Code No: R161203 **R16** SET - 1

I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2019
MATHEMATICS-III
(Com to AE.AME,CE,CSE,IT,EIE,EEE,ME,ECE,Metal E, Min E, E Com E, Agri E, Chem E, PCE,PE)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \end{bmatrix}$. (2M)

b) What is the nature of the quadratic form $x^2+y^2+z^2-2xy$? (2M)

c) Write the physical significance of grad φ. (2M)

d) Find the area bounded by the upper half of the curve $r = a(1-\cos\theta)$. (2M)

e) Prove that the work done in moving an object from P_1 to P_2 in a conservative force field $\overline{F}$ is independent of the path joining the two points P_1 and P_2. (2M)

f) Show that $\int_0^1 \left(\log\frac{1}{x}\right)^{m-1} dx = \Gamma(m)$. (2M)

g) Prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A. (2M)

PART -B

2. a) Use Gauss Seidel method to solve 25x + 2y + 2z = 69, 2x + 10y + z = 63, x + y +z = 43. (6M)

b) Reduce the quadratic form $x^2+4y^2+z^2+4xy+6yz+2zx$ to canonical form by linear transformation. Also find signature and rank of the quadratic form. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}. (7M)

4. a) Trace the curve $x^3 + y^3 + 3axy = 0$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$ by changing into polar coordinates. (7M)

5. a) Express the integral $\int_0^\infty \frac{x^c}{c^x} dx$ in terms of Gamma function. (7M)

b) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}} dx = \frac{B(m,n)}{a^n(1+a)^m}$. (7M)
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6. a) Find the directional derivative of the function f = $x^2 - y^2 + 2z^2$ at the point P=(1,2,3) in the direction of the line PQ where Q = (5,0,4). (7M)

b) Prove that $\nabla \times \left(\frac{\overline{A} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{A}}{r^n} + \frac{n(\overline{r}.\overline{A})\overline{r}}{r^{n+2}}$. (7M)

7. If $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$, evaluate $\int_S \overline{F}.\overline{n}ds$ where S is the surface of the cube bounded by x = 0, x = a, y = 0, y = a, z = 0, z = a. (14M)

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PART –A

1. a) Find the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \\ 3 & 0 & 4 \end{bmatrix}$ by reducing it into echelon form. (2M)

b) What is the nature of the quadratic form $x^2-2y^2+z^2-2zy$? (2M)

c) Evaluate $\int_0^1 \sqrt[3]{\log\frac{1}{x}}\,dx$. (2M)

d) If λ is eigenvalue of an orthogonal matrix, then show that $\frac{1}{\lambda}$ is also an eigenvalue. (2M)

e) Find the area bounded by the curves $y = x$ and $y = x^2$. (2M)

f) In what direction from the point $(1,-1,3)$ the directional derivative of $\phi = 2xy + z^2$ is maximum? What is the magnitude of this maximum? (2M)

g) State Gauss divergence theorem. (2M)

PART –B

2. a) Solve by Gauss – Seidal method, the equations. (6M)

$$9x - 2y + z - t = 50$$
$$x - 7y + 3z + t = 20$$
$$-2x + 2y + 7z + 2t = 22$$
$$x + y - 2z + 6t = 18$$

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix}$ and find A^{-1}. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$. (7M)

b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to orthogonal transformation. Also find signature and rank of the quadratic form. (7M)

4. a) Trace the curve $y^2(a+x) = x^2(a-x)$. (7M)

b) By changing the order of integration, evaluate $\int_0^1\int_1^{2-x} xy\,dx\,dy$. (7M)
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5. a) Evaluate $\int_0^1 (8-x^3)^{1/3}dx$ using β and γ functions. (7M)

   b) Prove that $\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\ \Gamma(2n+1)}{2^{2n}\Gamma(n+1)}$. (7M)

6. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$. (7M)

   b) Prove that curl $(\bar{a}\times\bar{b}) = \bar{a}div\bar{b} - \bar{b}div\bar{a} + (\bar{b}.\nabla)\bar{a} - (\bar{a}.\nabla)\bar{b}$. (7M)

7. Verify Stoke's theorem for $\bar{F} = (2x-y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$ over the upper half of surface of sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy- plane. (14M)

Code No: R161203 **R16** SET - 3

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 7 & 8 & 1 \\ 1 & 3 & 4 & 2 \\ 3 & 5 & 6 & 10 \end{bmatrix}$. (2M)

b) What is the nature of the quadratic form $-2x^2+2y^2-z^2-2xy$? (2M)

c) Find the complete area of the curve $a^2y^2 = x^3(2a-x)$. (2M)

d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4\theta\cos^2\theta d\theta$. (2M)

e) If λ is an eigenvalue of a nonsingular matrix A, then show that $\frac{|A|}{\lambda}$ is an eigenvalue of adj A. (2M)

f) In what direction from the point (2, –1, 1) the directional derivative of $\phi = xy^2 + yz^3$ is maximum. What is the magnitude of this maximum? (2M)

g) State Stoke's theorem. (2M)

PART -B

2. a) Apply Guass – Seidel method to solve the equations. (6M)

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & -1 \\ 1 & -2 & 10 \end{bmatrix}$ and find A^{-1}. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

b) Reduce the quadratic form $4x^2+3y^2+z^2-8xy-6yz+4zx$ to orthogonal transformation. Also find signature and rank of the quadratic form. (7M)

4. a) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$. (7M)

b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)dxdy$. (7M)
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5. a) Evaluate $\int_0^\infty \frac{x^2}{1+x^4}dx$ using β and γ functions. (7M)

b) Show that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. (7M)

6. a) Find the angle between the normal to the surface $x^2 = yz$ at the points (1, 1, 1) and (2, 4, 1). (7M)

b) Find the constants a, b, c so that $(x+2y+az)\bar{i}+(bx-3y-z)\bar{j}+(4x+cy+2z)\bar{k}$ is irrotational. Also find the scalar potential. (7M)

7. Verify Green's theorem for $\int_C (xy+y^2)dx+(x^2)dy$ where C is the curve bounded by y = x^2 and y = x. (14M)

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 2 & 6 & 8 & 2 \\ 1 & 3 & 4 & 1 \\ 3 & 5 & 6 & 10 \end{bmatrix}$ (2M)

b) What is the nature of the quadratic form x^2-$3y^2$-z^2-zy ? (2M)

c) Prove that zero is an eigen value of a matrix if and only if it is singular. (2M)

d) In what direction from the point $(1,-2,-1)$ the directional derivative of $\phi = x^2yz + 4xz^2$ is maximum? What is the magnitude of the maximum? (2M)

e) Show that in an irrotational field, the value of a line integral between two points A and B will be independent of the path of integration and be equal to their potential difference. (2M)

f) Find the area bounded by the curves $y = x^2$ and $x = y^2$. (2M)

g) Show that $\int_0^\infty x^{n-1}e^{-kx}dx = \frac{\Gamma(n)}{k^n}$ (2M)

PART -B

2. a) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ and find A^{-1}. (6M)

b) Apply Guass – Seidel method to solve the equations (8M)

$$20x + y - 2z = 17,$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$. (7M)

b) Using Lagrange's reduction, transform $x_1^{\,2} + 2x_2^{\,2} - 7x_3^{\,2} - 4x_1x_2 + 8x_1x_3$. (7M)

4. a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (7M)

b) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dydx.$ (7M)
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5. a) Show that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. (7M)

b) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx$ in terms of Gamma function. (7M)

6. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point (4, -3, 2). (7M)

b) Prove that $grad(\overline{a}.\overline{b}) = (\overline{b}.\nabla)\overline{a} + (\overline{a}.\nabla)\overline{b} + \overline{b}\times curl\overline{a} + \overline{a}\times curl\overline{b}$. (7M)

7. Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, over the cube formed by the planes x = 0, x = a, y = 0, y = b, z = 0, z = c. (14M)